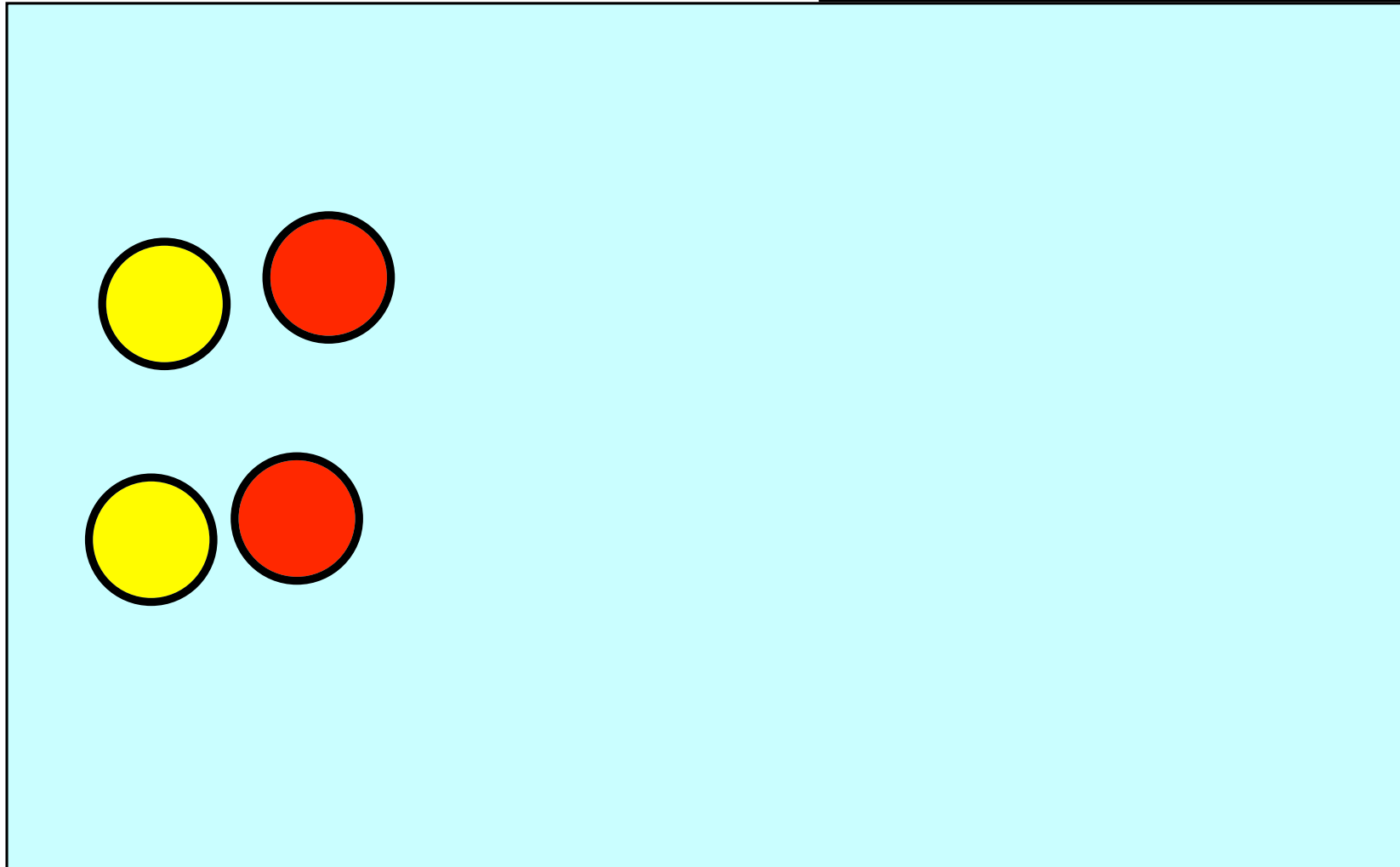
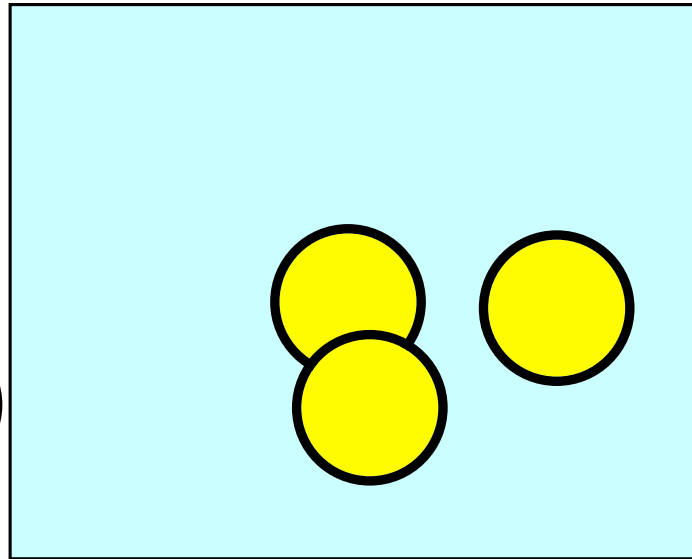
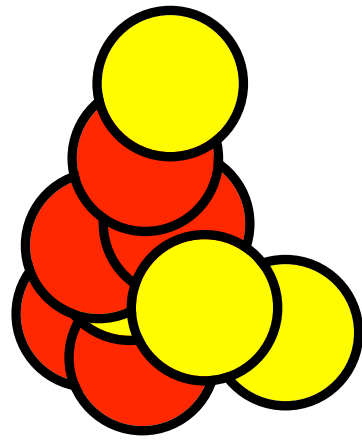


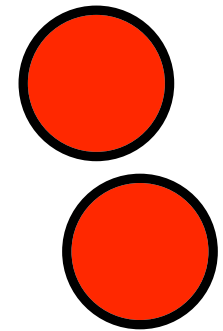
 Boys

 Girls

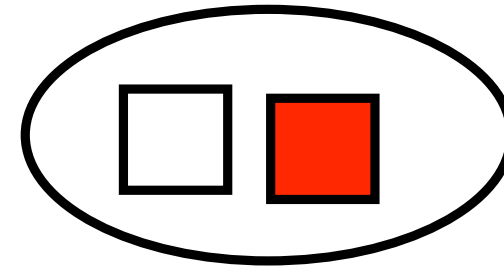
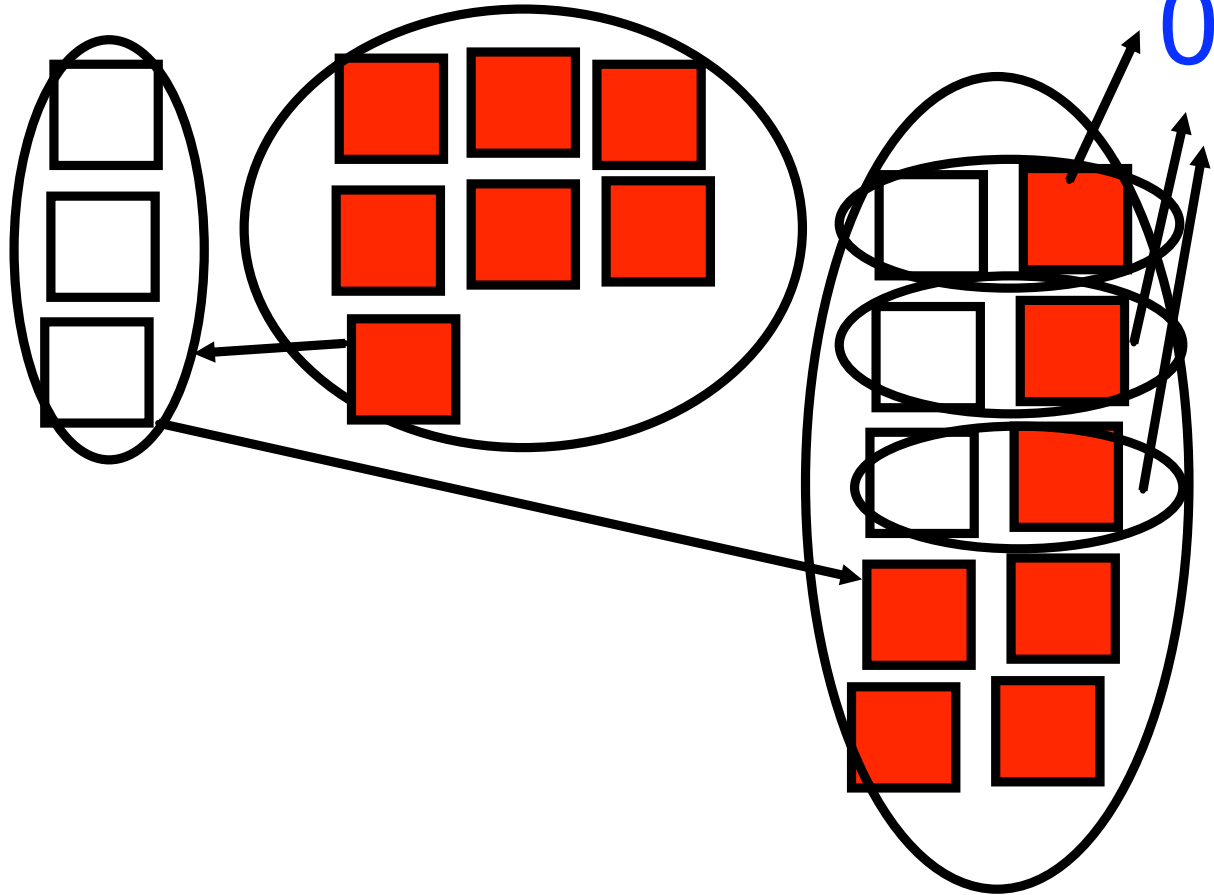


$$(-2) - (-4) = +2$$

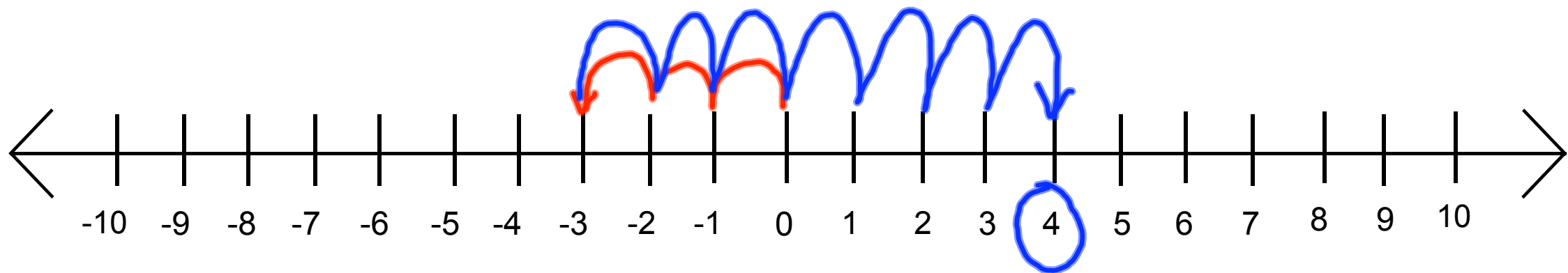
$$(+2) - (+5) = (-3)$$



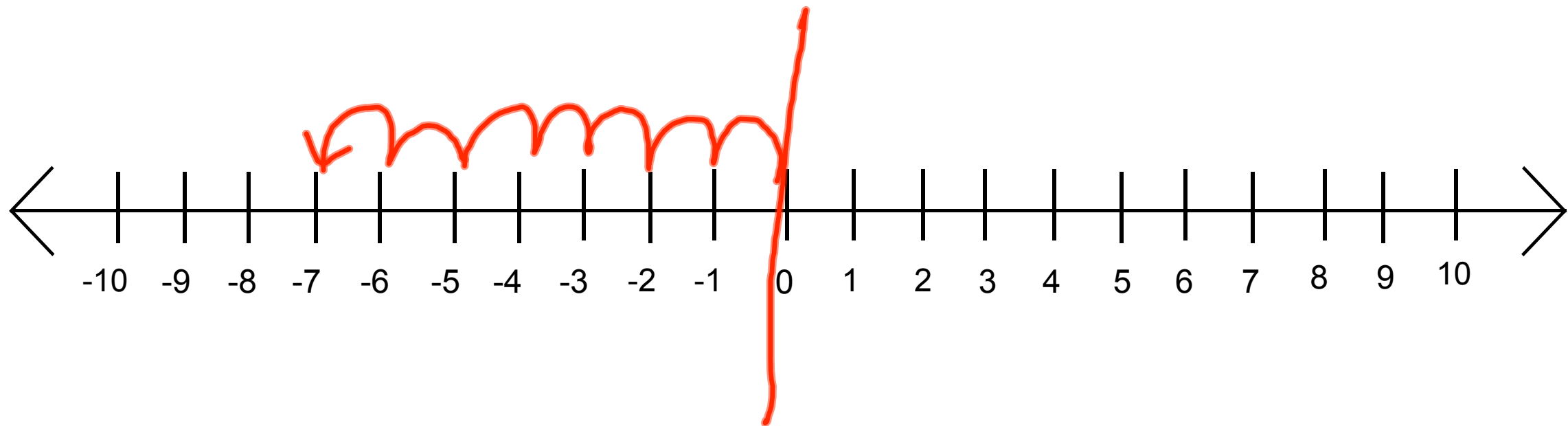
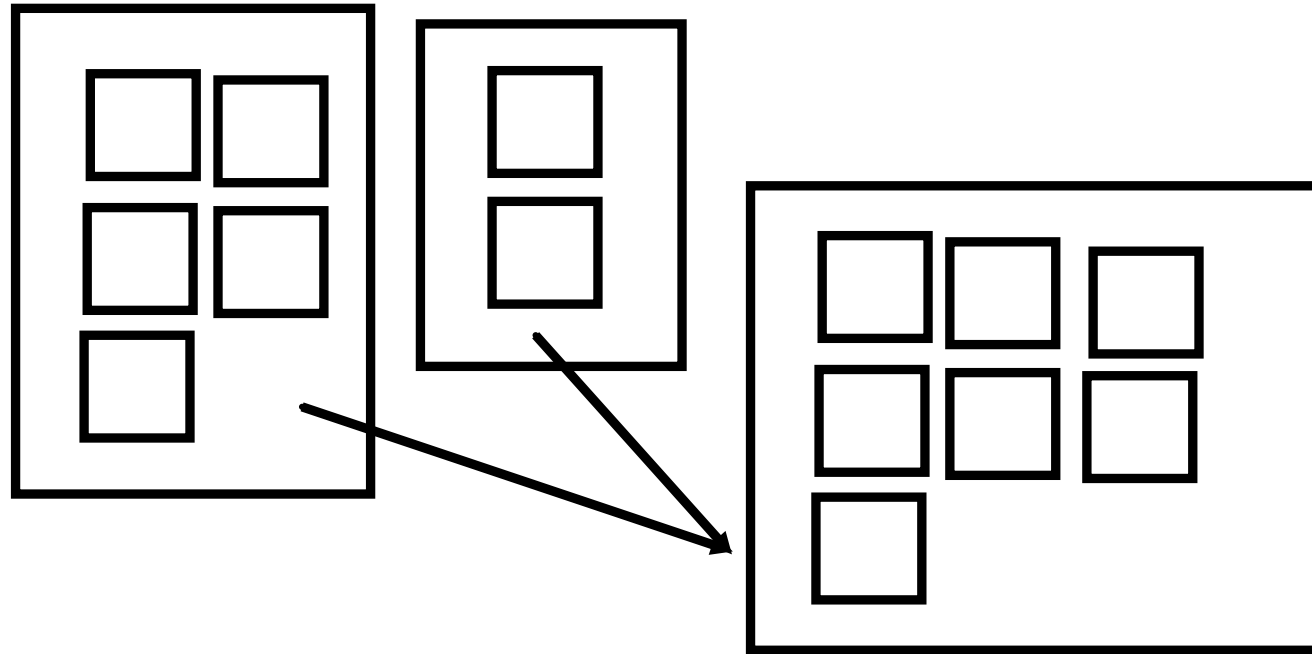
$$(-3) + (+7) = +4$$



the Zero Model
 $(+1) + (-1) = 0$

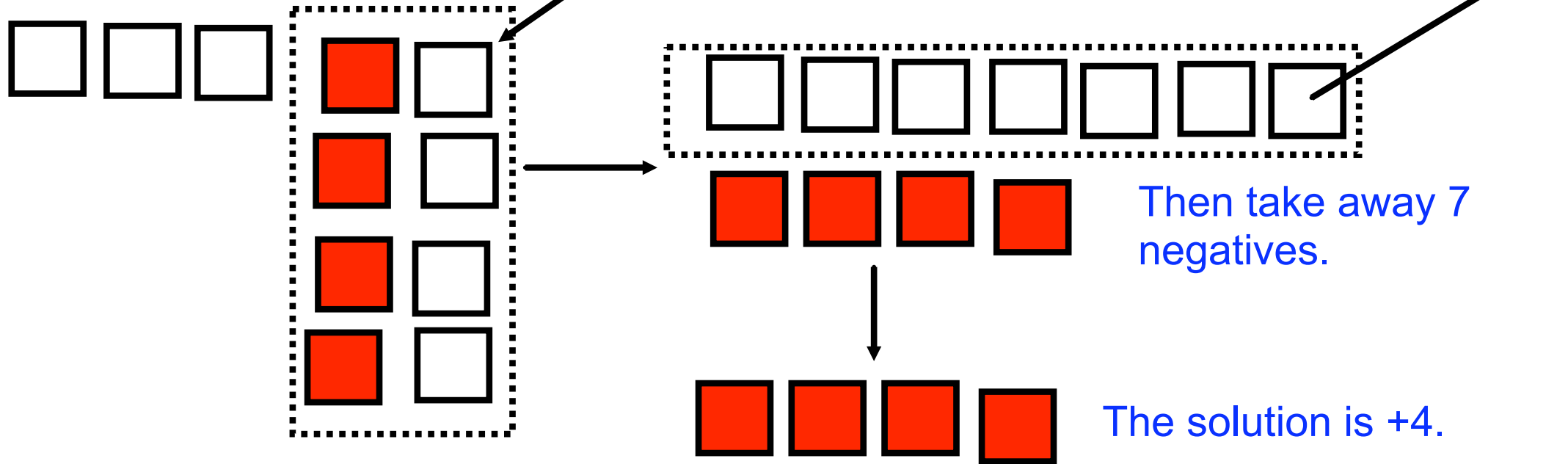


$$(-5) + (-2) = -7$$

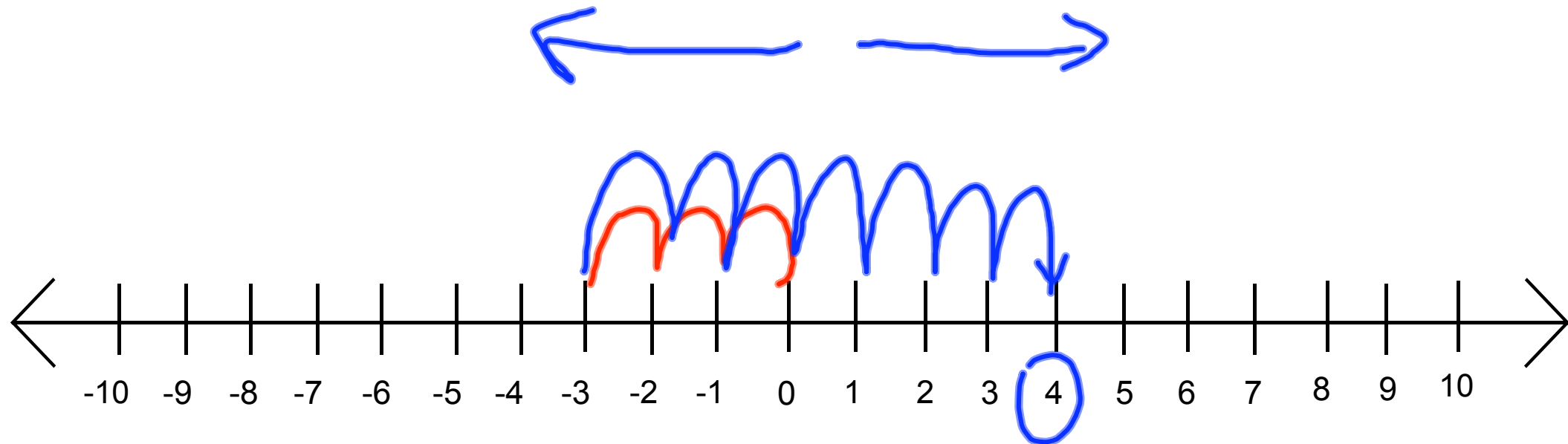


$$(-3) - (-7)$$

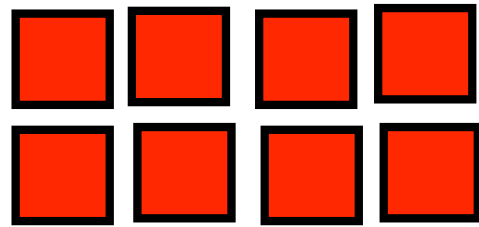
Add zeros until I have enough tiles to take away 7 negatives.



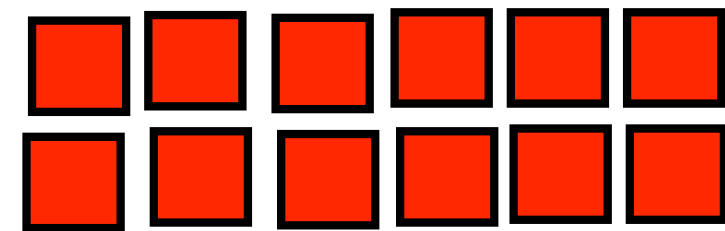
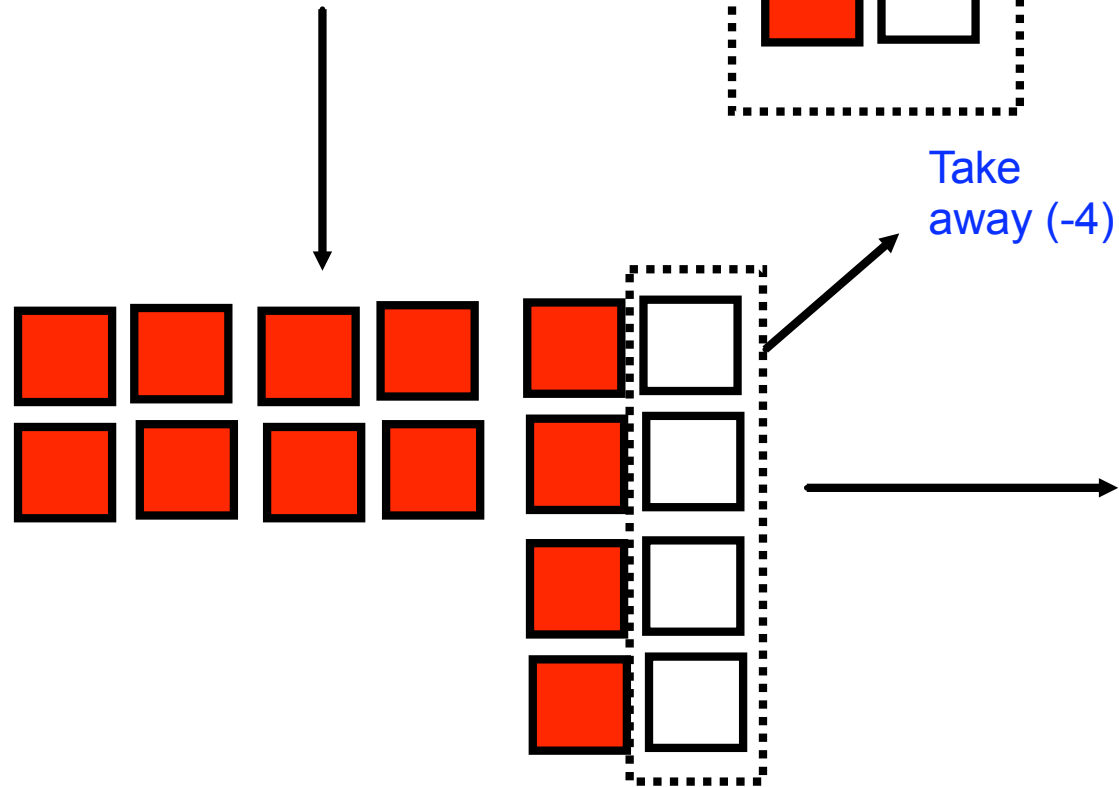
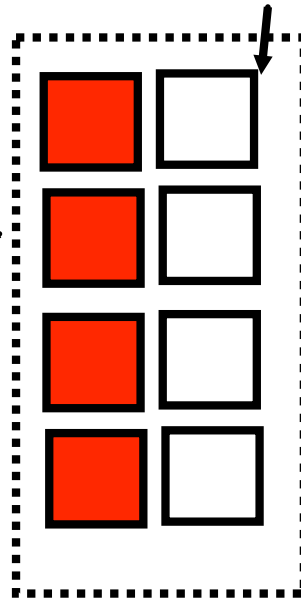
$$(-3) - (-7) = +4$$



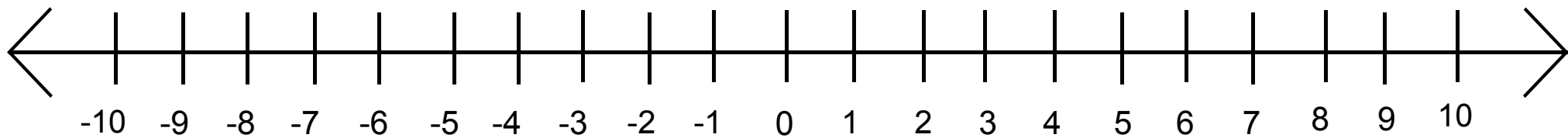
$$(+8) - (-4)$$



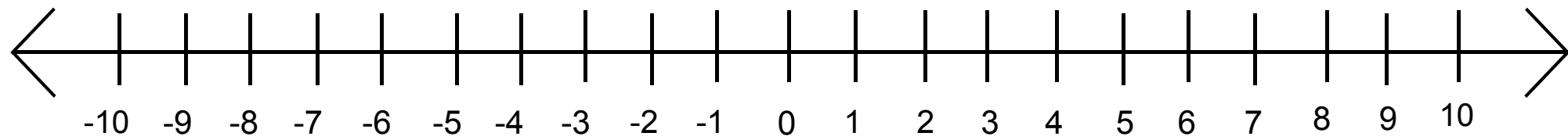
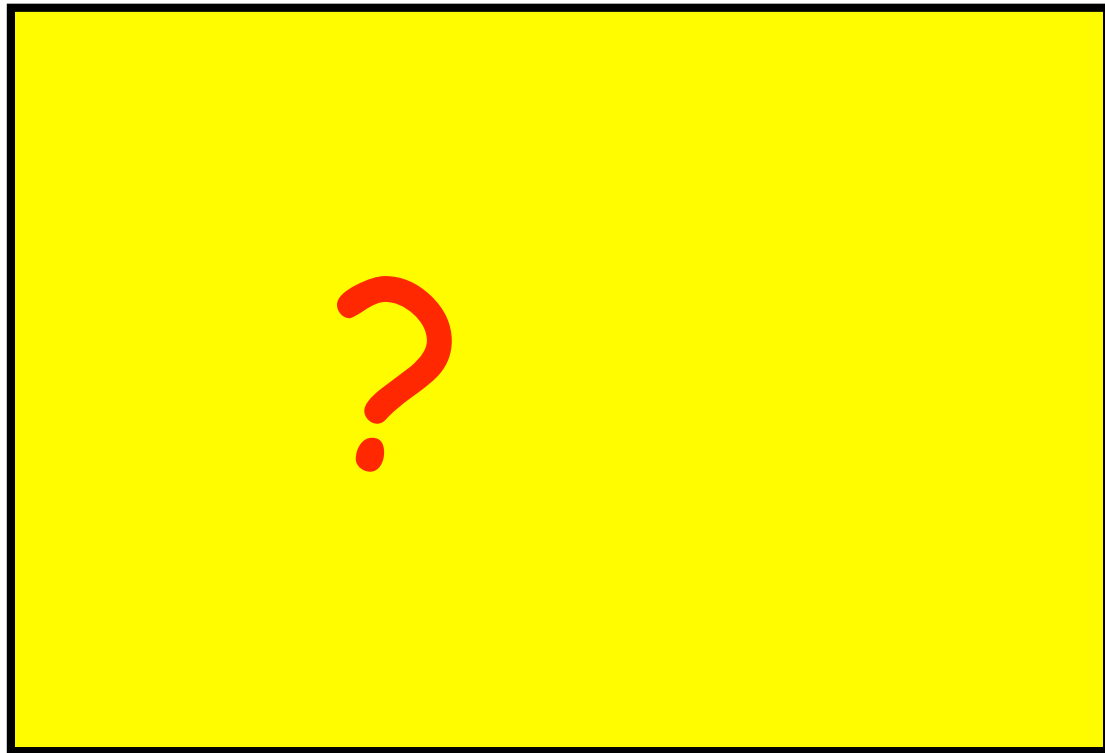
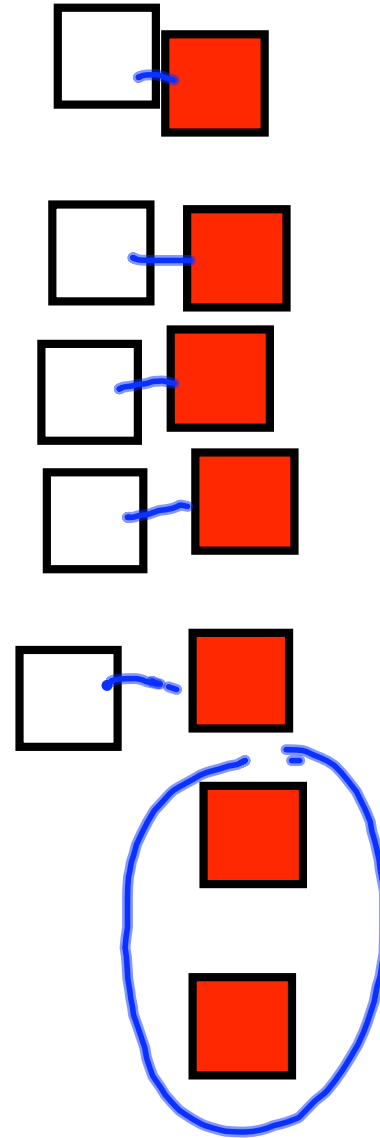
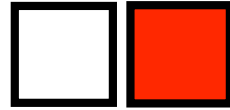
Add zeros until I have enough tiles to take away 4 negatives.



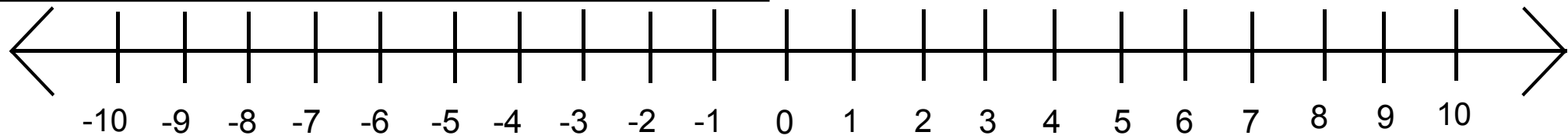
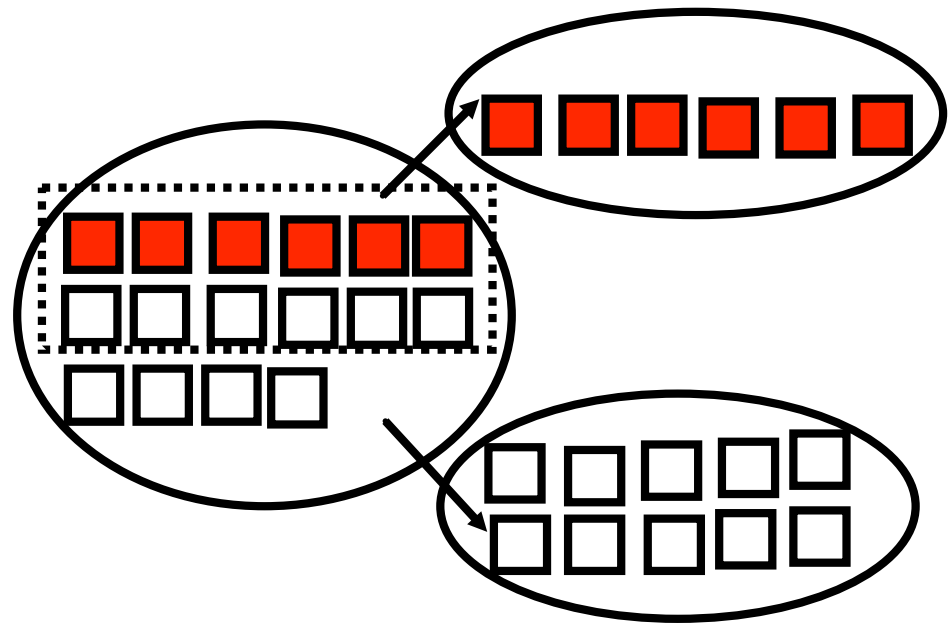
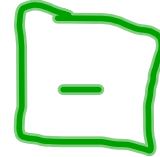
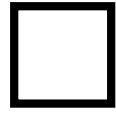
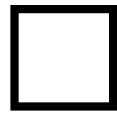
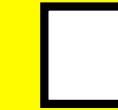
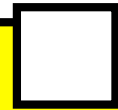
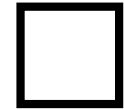
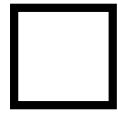
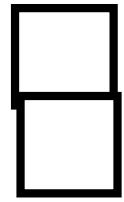
$$(+8) - (-4) = +12$$



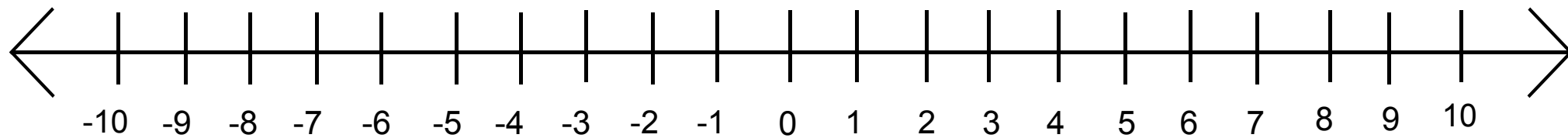
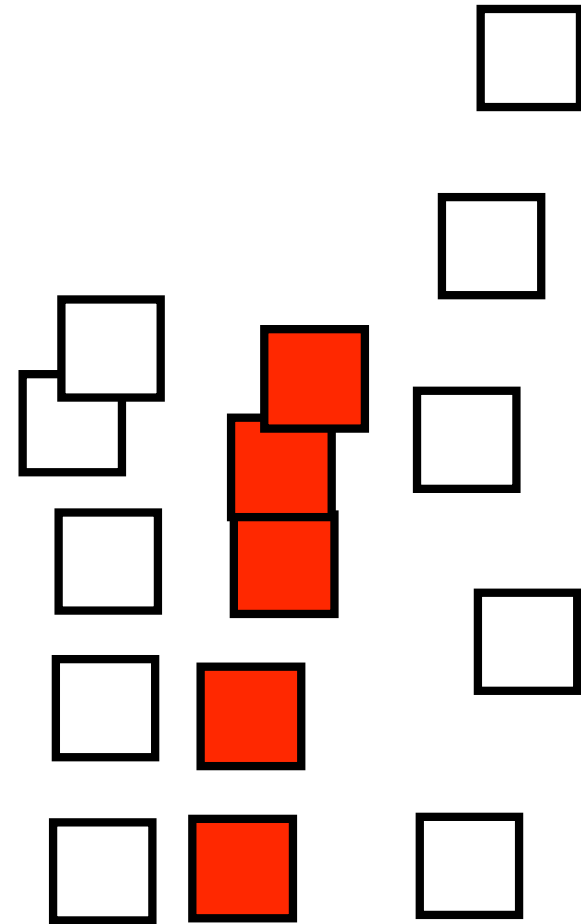
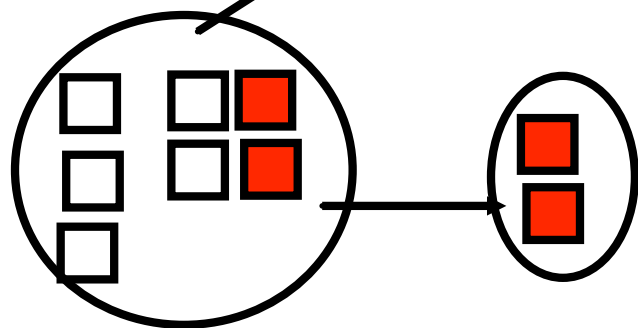
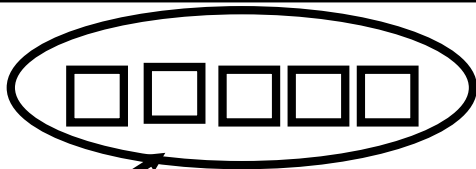
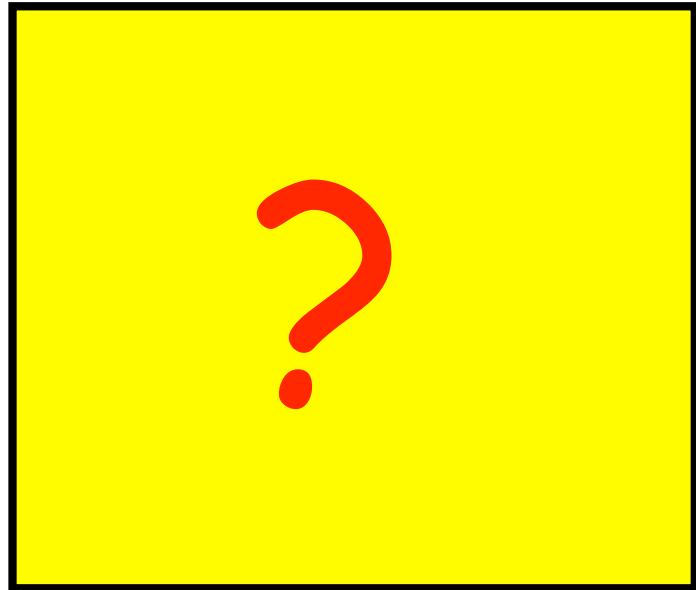
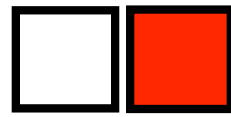
$$(-5) + (+7) = \boxed{?}$$



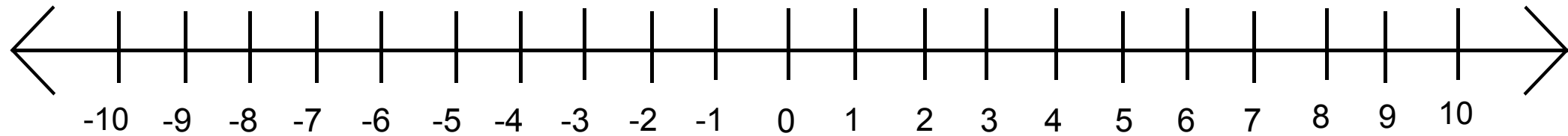
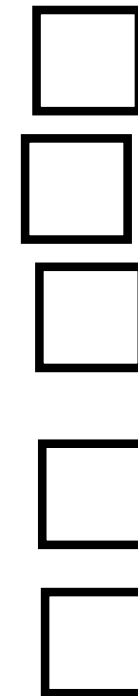
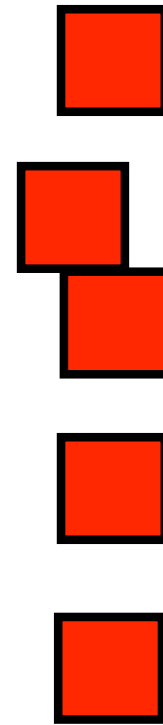
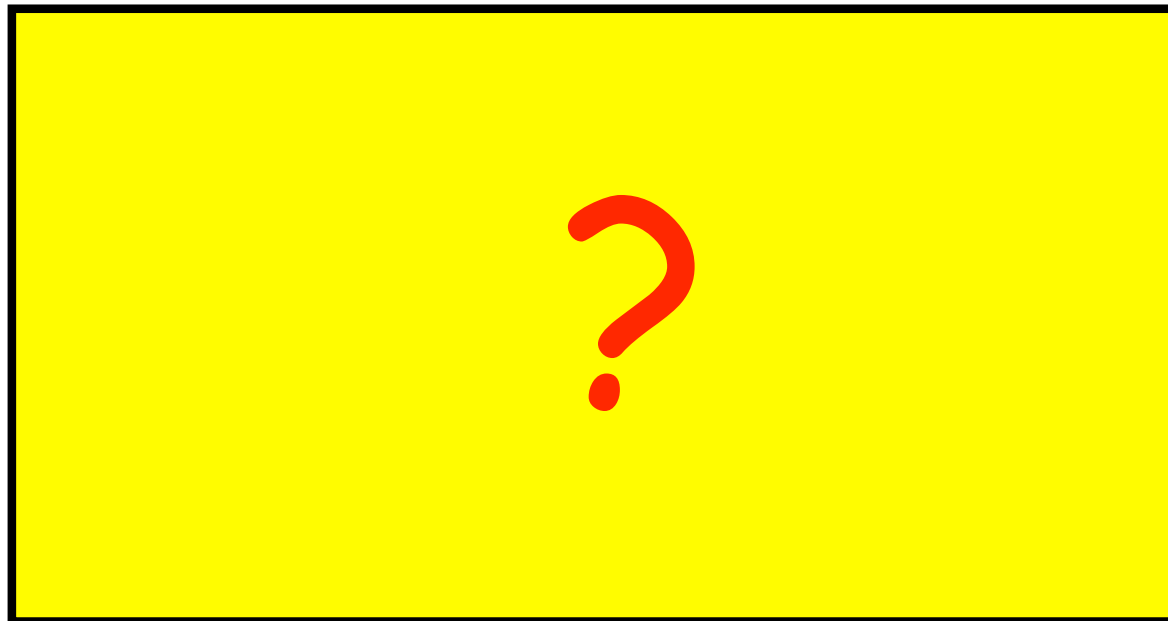
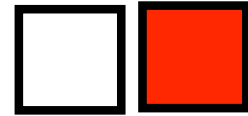
$$(-4) - (+6) = \boxed{?}$$



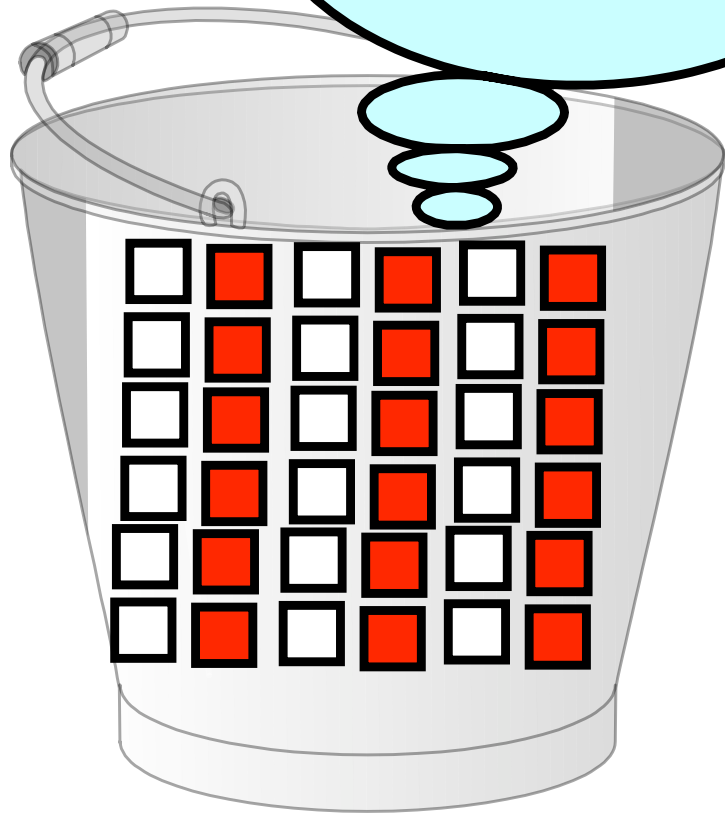
$$(-3) - (-5) = \boxed{?}$$



$$(+7) - (-5) = \boxed{?}$$



Always start with a bucket of zero and some extra zeros to use.

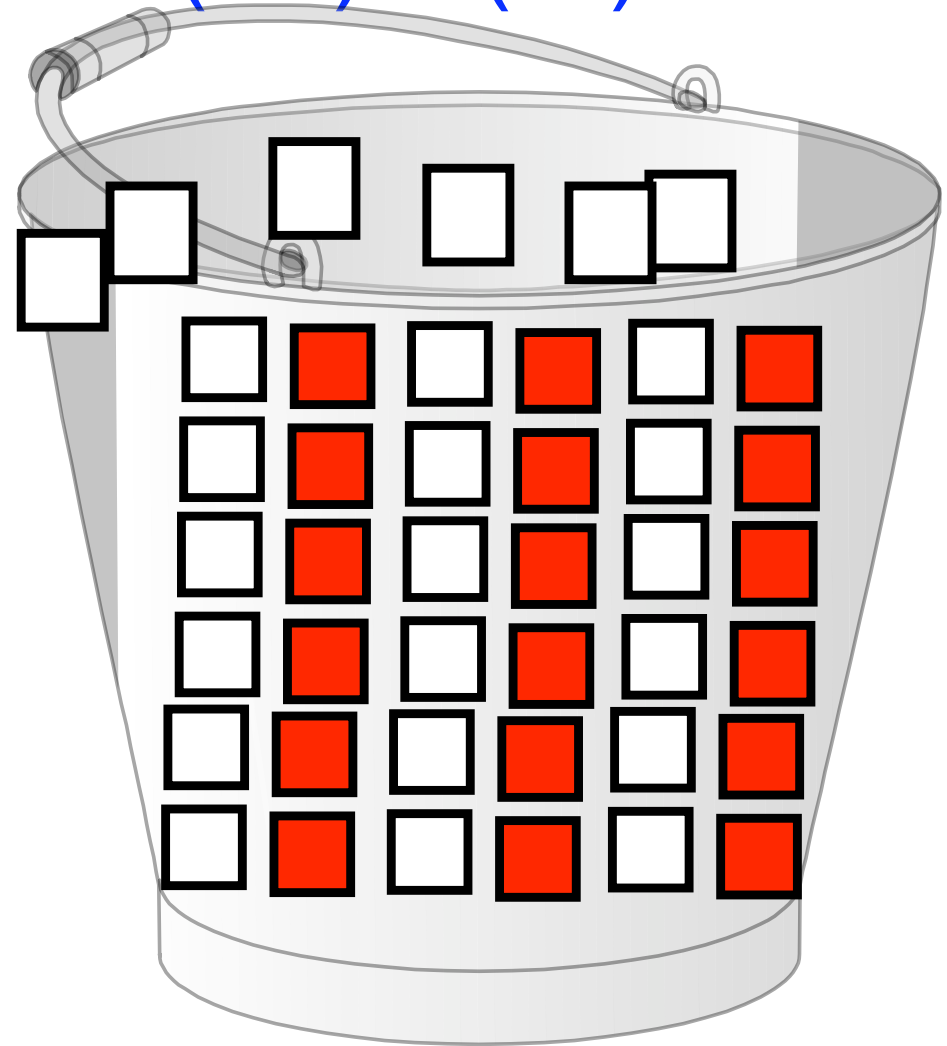


Say it in words:

Add on 3 sets of -2



$$(+3) \times (-2) = -6$$

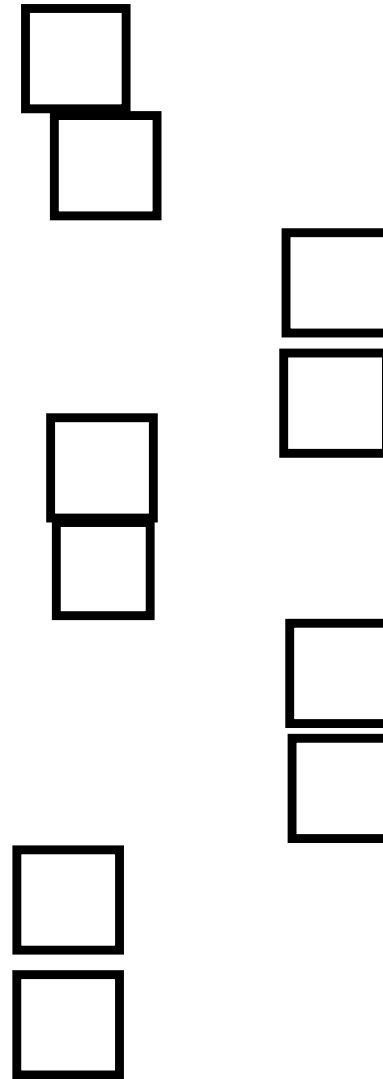
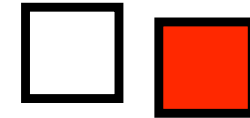
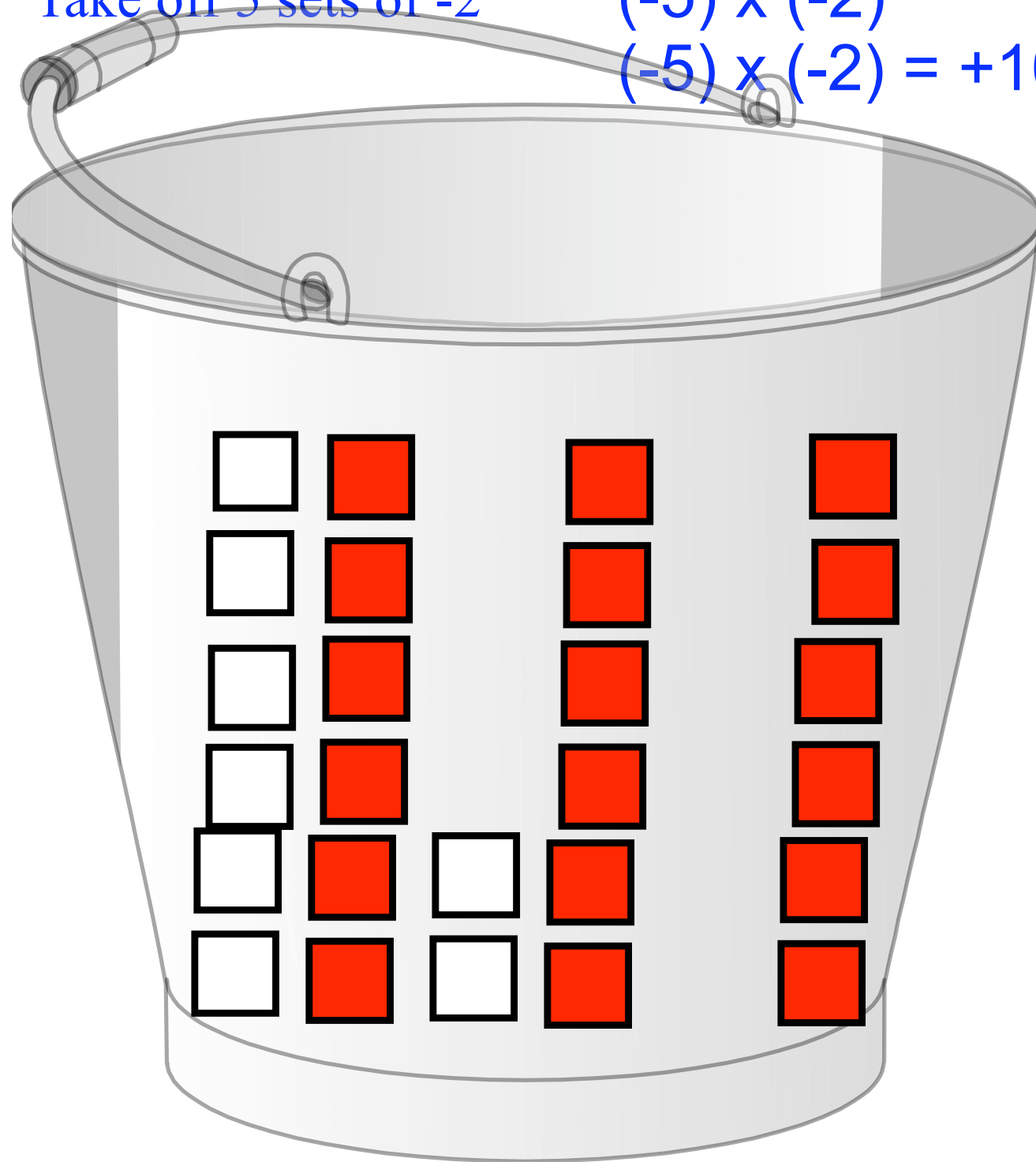


Multiplying Integers

Say it in words:

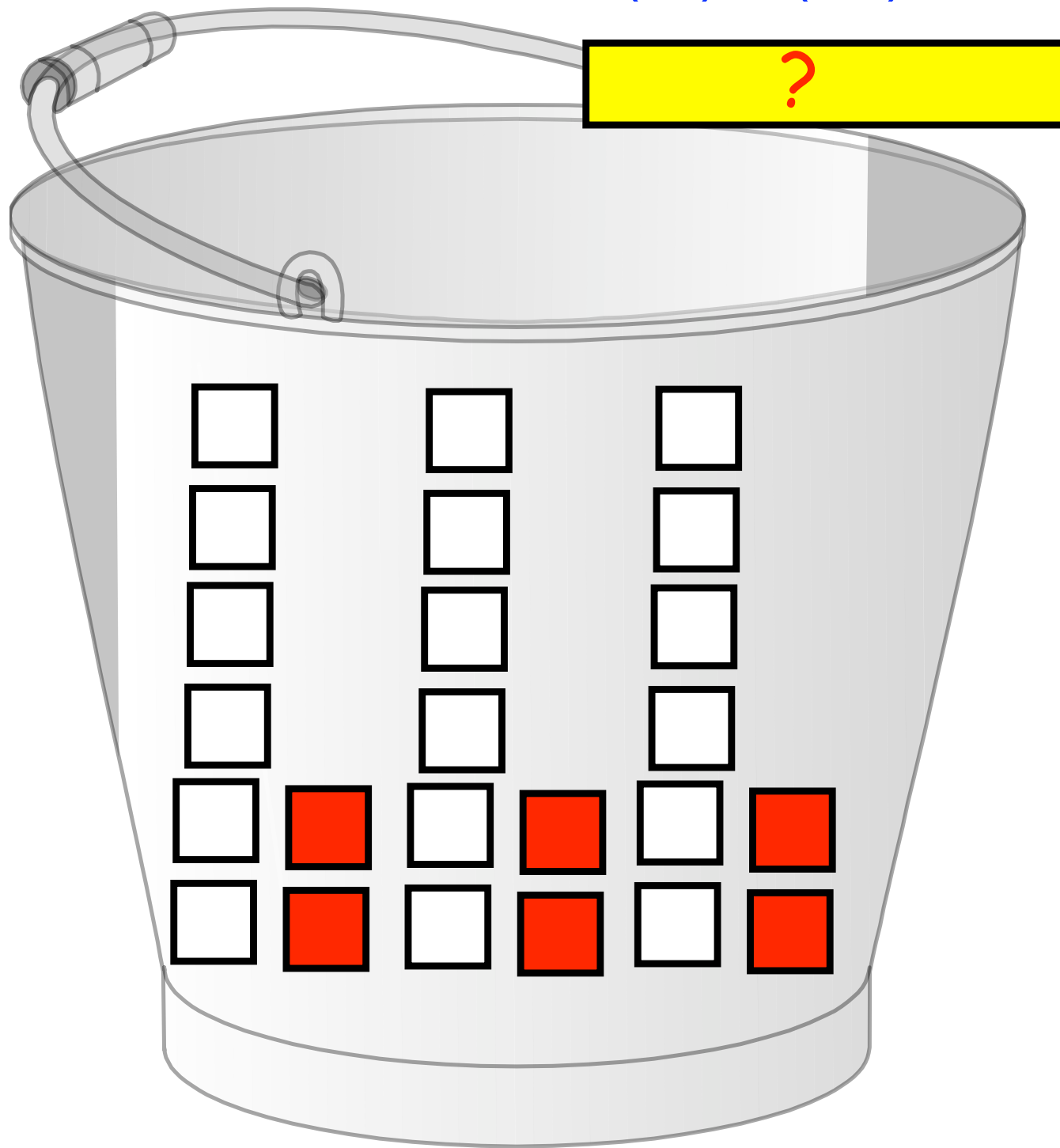
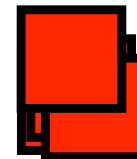
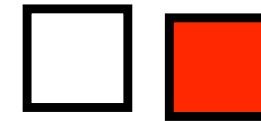
Take off 5 sets of -2

$$\begin{aligned} &(-5) \times (-2) \\ &(-5) \times (-2) = +10 \end{aligned}$$



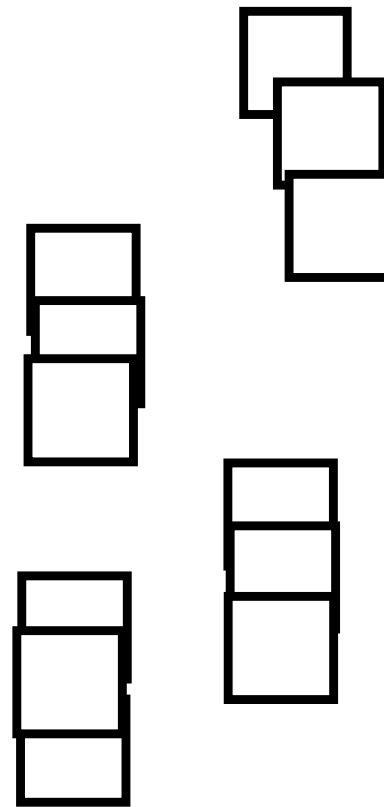
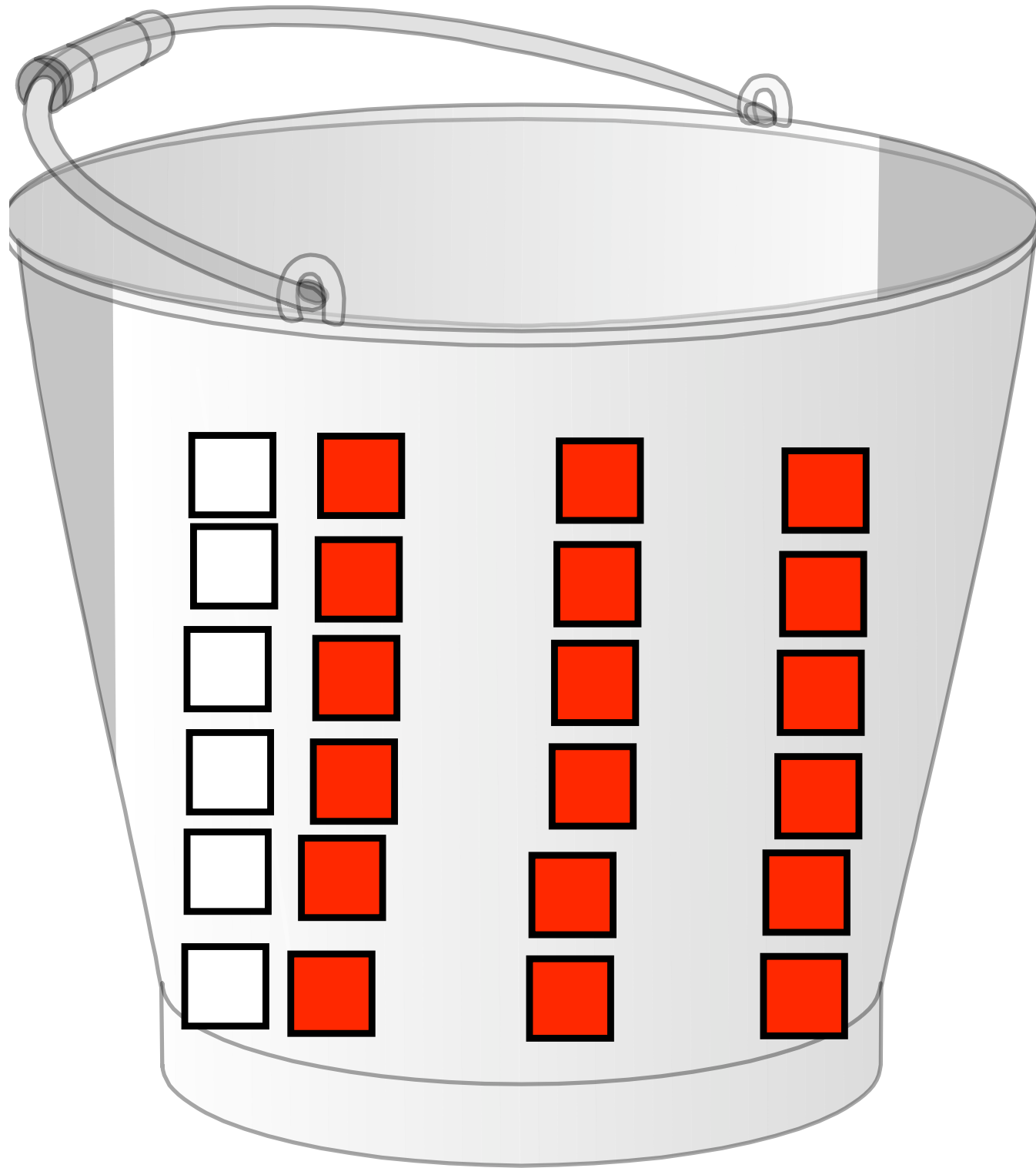
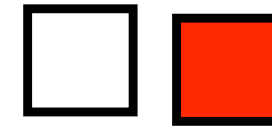
Take off 3 sets of +4

$$\begin{aligned} &(-3) \times (+4) \\ &(-3) \times (+4) = -12 \end{aligned}$$



Take off 4 sets of -3

$$(-4) \times (-3)$$

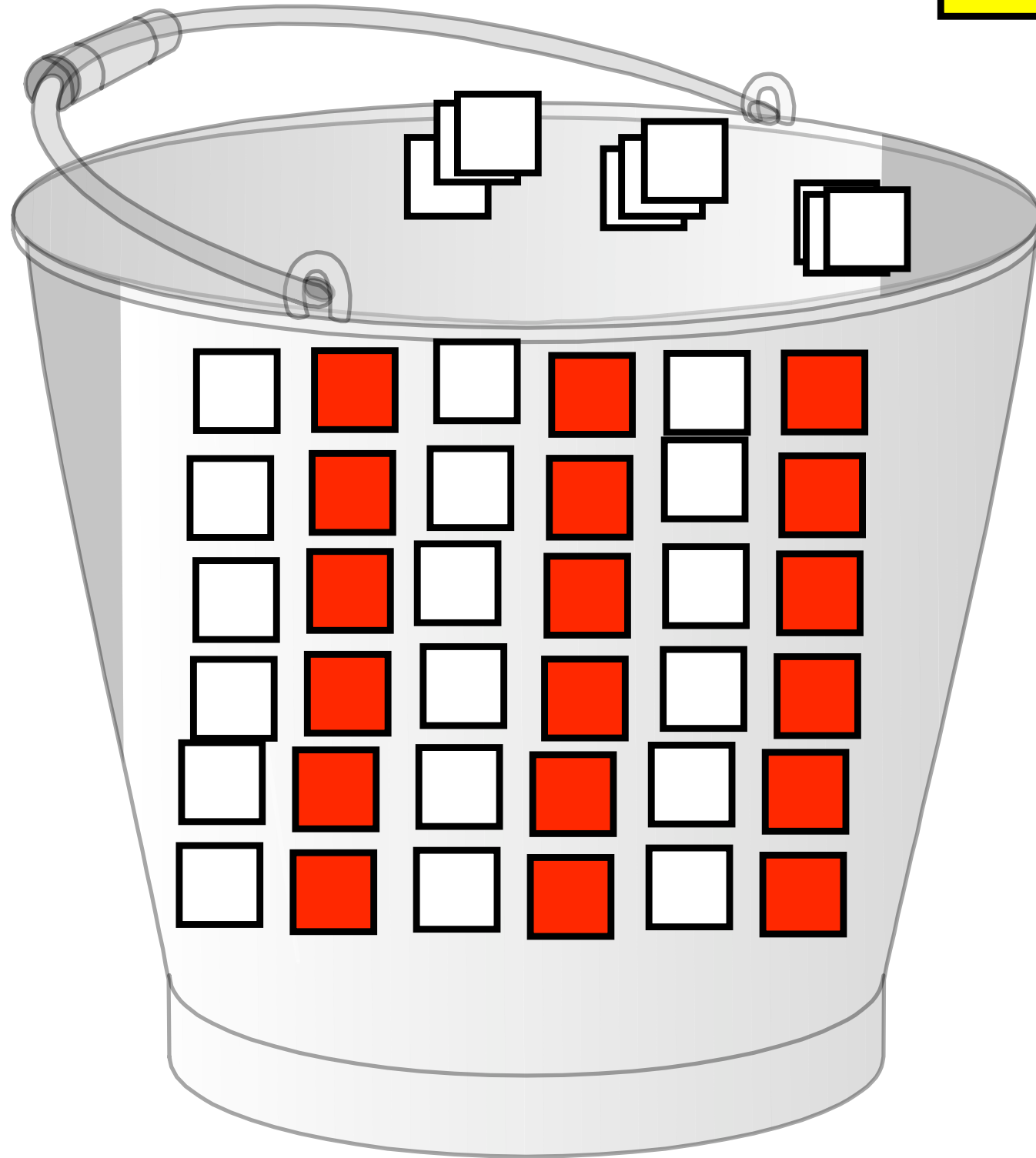


Say it in words:

Add on 3 sets of -3

$$(+3) \times (-3)$$

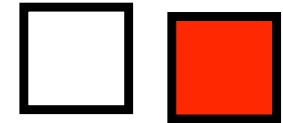
$$(+3) \times (-3) = -9$$



Say it in words: Take off 4 sets of +3

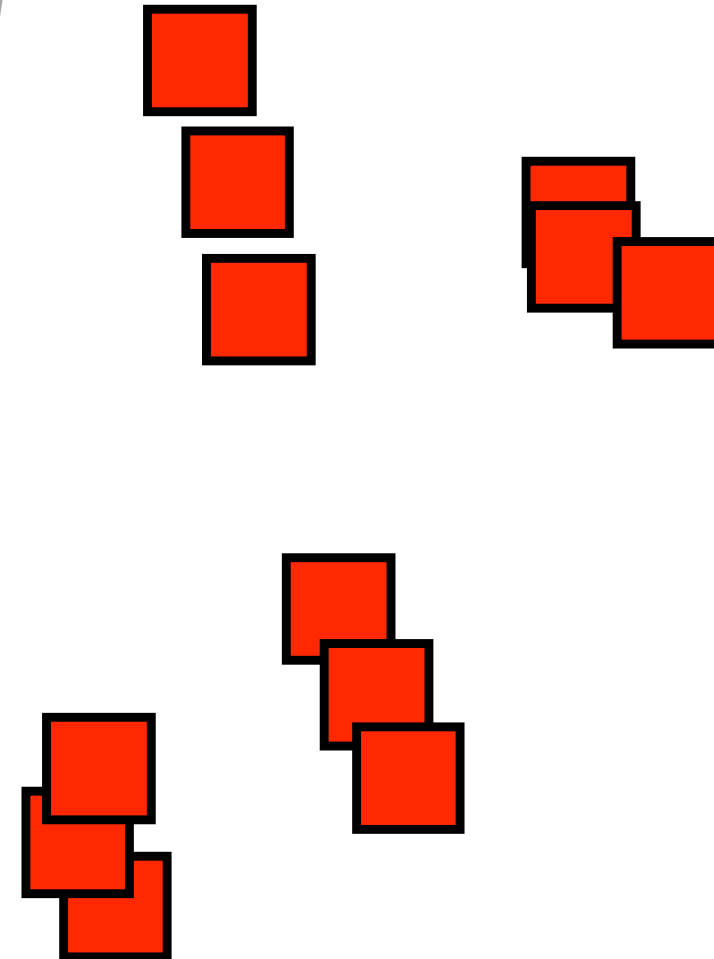
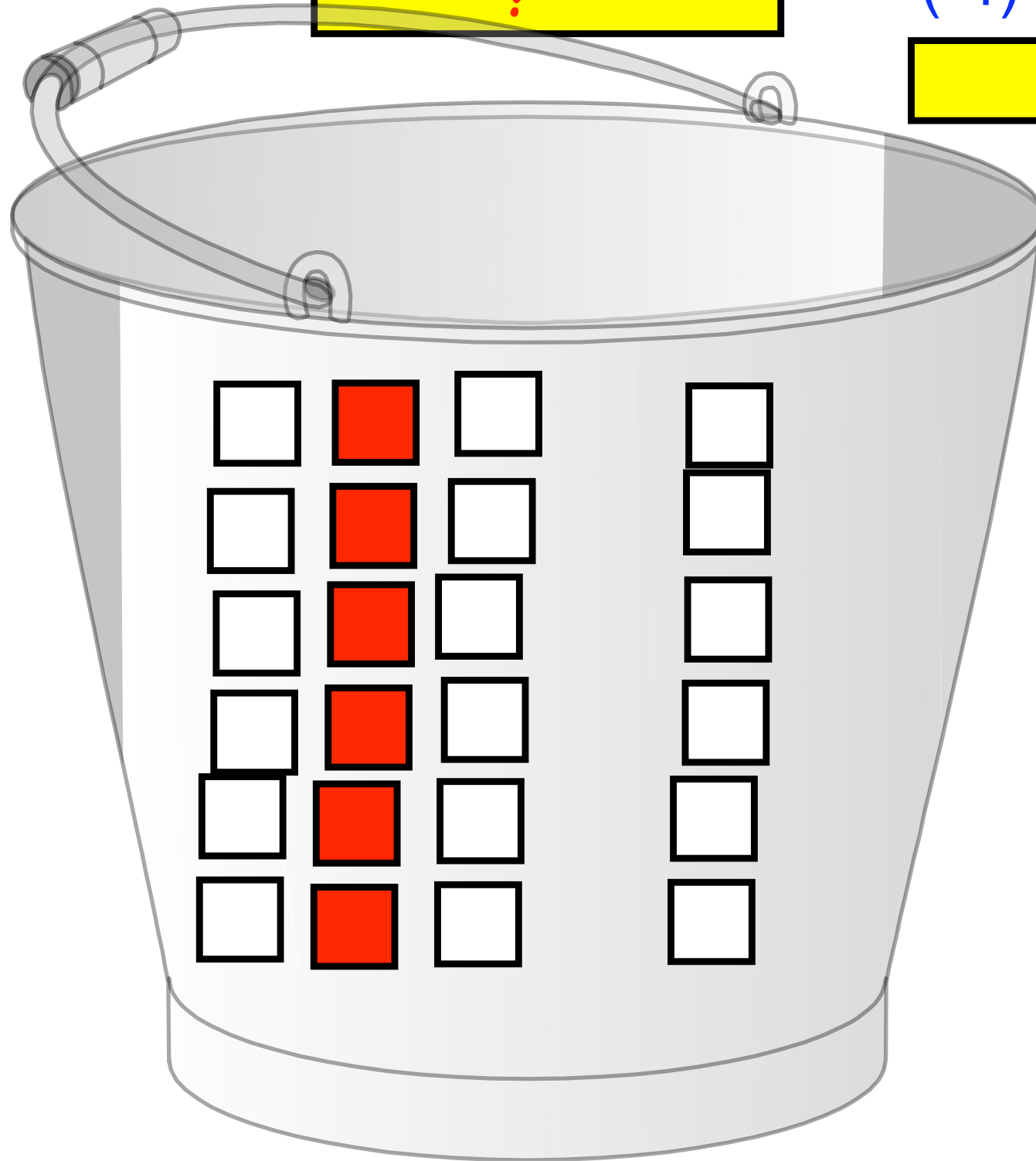
$$(-4) \times (+3)$$

$$(-4) \times (+3) = -12$$



?

?

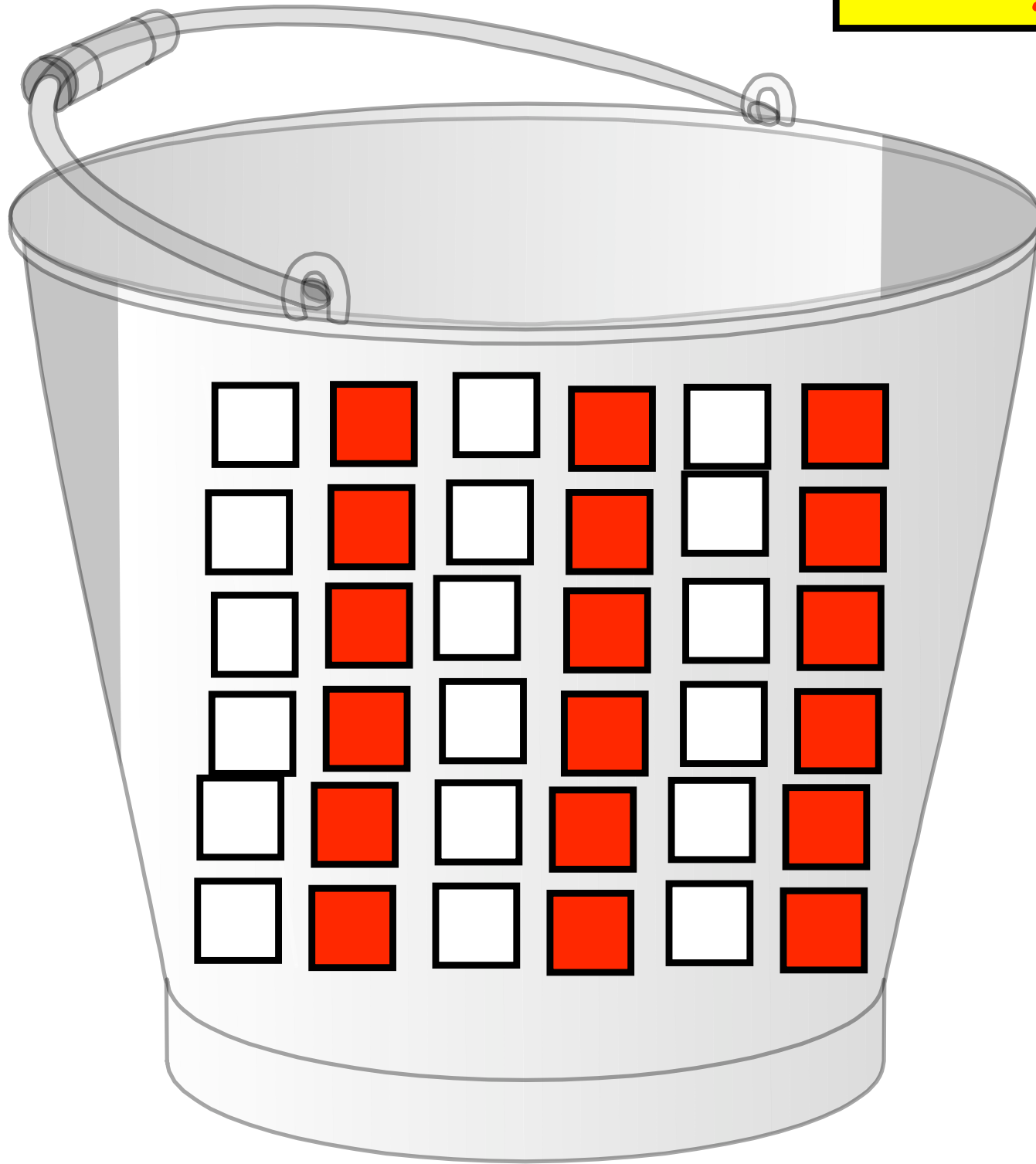
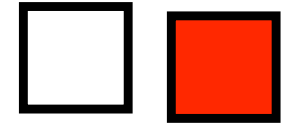


Say it in words:

?

$$(-5) \times (-3)$$

?



$$-n x + m$$

$$\hookrightarrow -$$

$$+n x - m$$

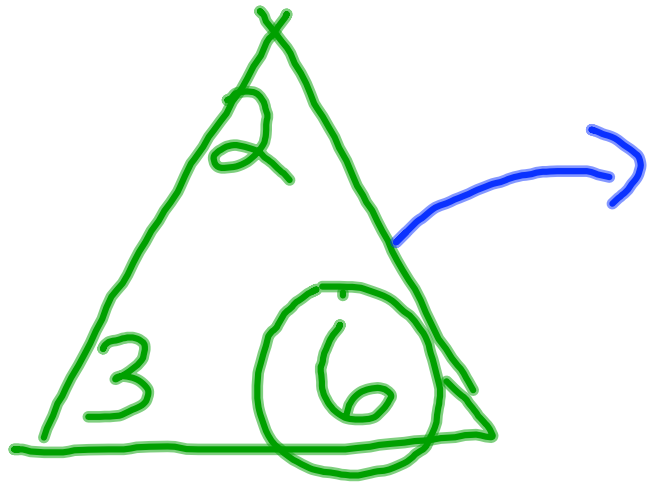
$$\hookrightarrow -$$

$$+n x + m$$

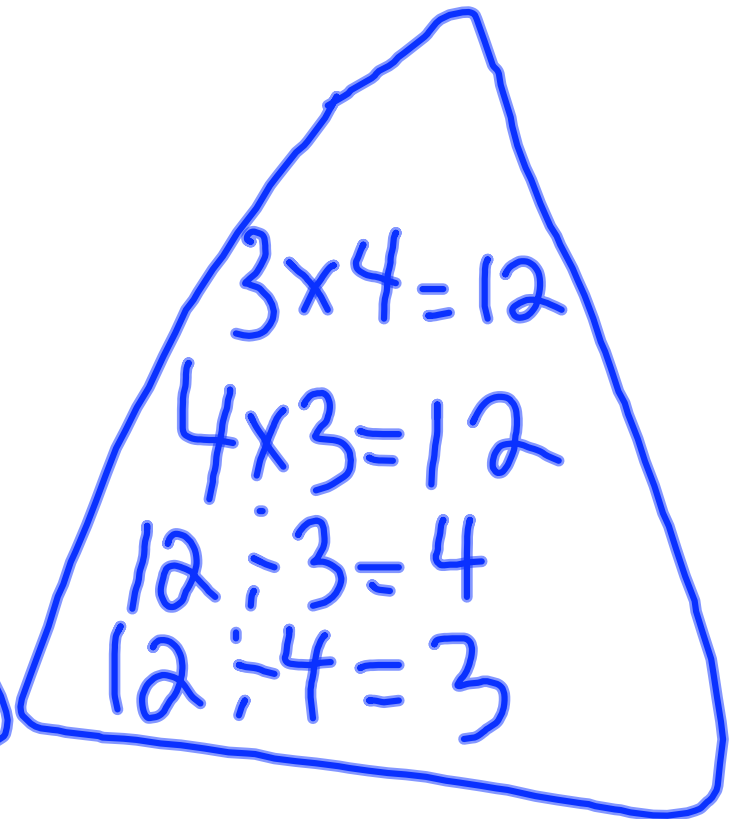
$$\hookrightarrow +$$

$$-n x - m$$

$$\hookrightarrow +$$



$$\begin{aligned}2 \times 3 &= 6 \\3 \times 2 &= 6 \\6 \div 2 &= 3 \\6 \div 3 &= 2\end{aligned}$$



$$-2 \times +4 = -8$$

Test

$$+4 \times -2 = -8$$

Commutativity

$$-8 \div +4 = -2 \quad -12 \div -3 = \underline{\quad}$$

$$-8 \div -2 = +4 \quad -3 \times \underline{\quad} = -12$$

Diving Integers!

Consider, if $(+2) \times (-4) = (-8)$ means add two sets of -4 to zero and you get -8, then what does it mean to divide?

We know, because it is in the fact family, that the following is true, but what does this tell us?

$$(-8) \div (-4) = (+2)$$

I want to make a group of -8 using sets of -4.

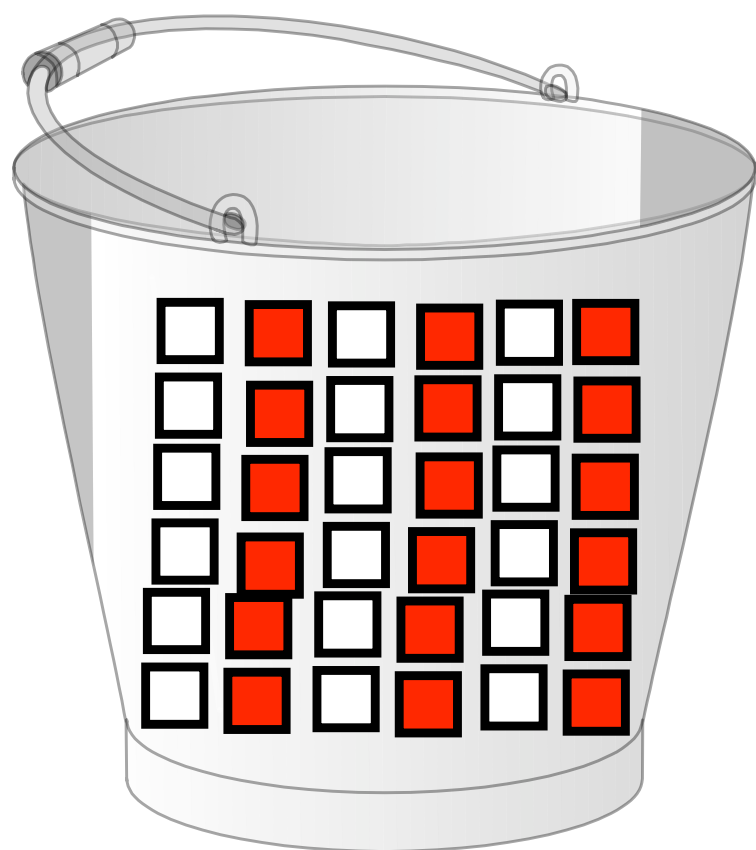
How many sets of -4 do I need to make -8? Will I add the sets of -4 to the bucket of zero or take them from the bucket of zero?

I would need to **add two (+2)** sets of -4 to a bucket of zero to make -8. So my solution is +2 meaning add two sets of -4 to the bucket of zero.

$$(+9) \div (+3) = ?$$

Think:

How many sets of +3 do I need to make +9?
Will I be adding sets of +3 to the bucket of zero (+ answer) or taking sets of +3 from the bucket of zero (- answer)?

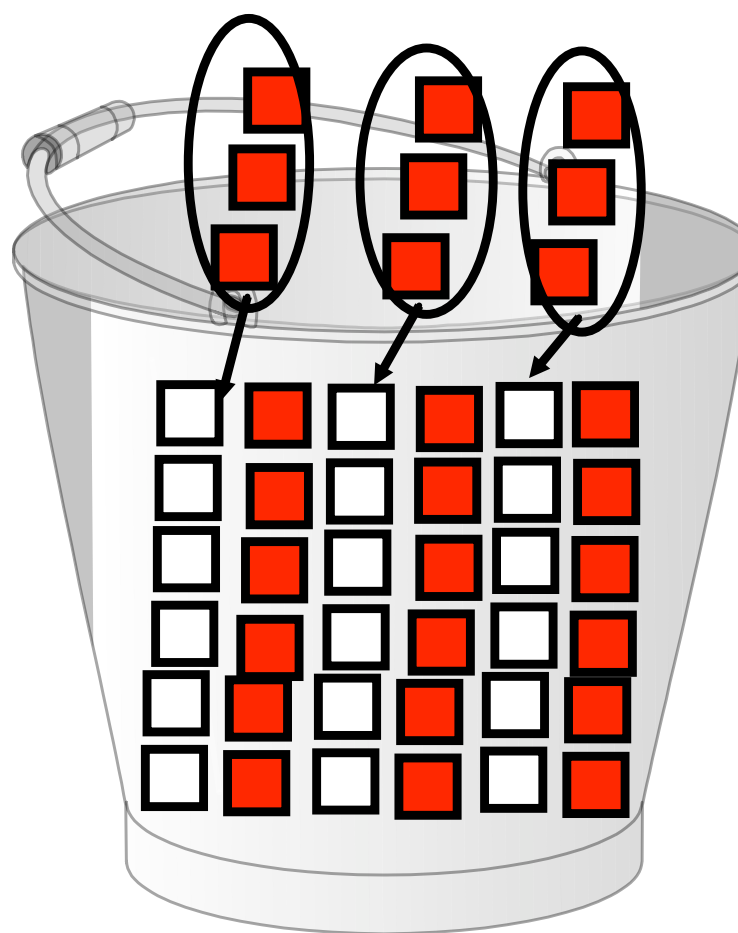


$$(+9) \div (+3) = (+3)$$



Solution process:

To make the value of the bucket +9 I would need to add three sets of +3, so my solution must be +3.



$$(+8) \div (-4) = ? \quad -2$$

$$(+8) \div (-4) = (-2)$$

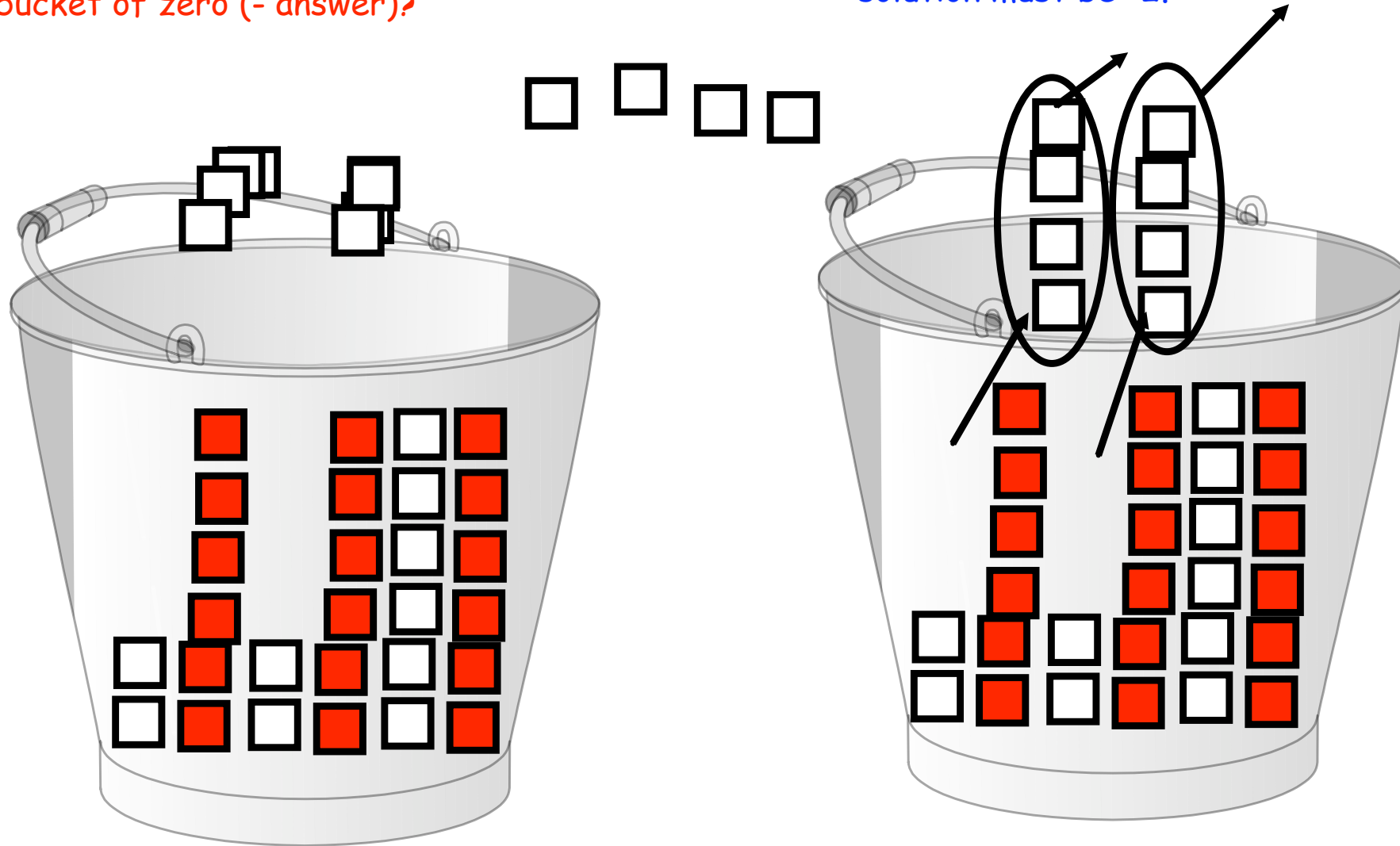


Think:

How many sets of -4 do I need to make +8?
Will I be adding sets of -4 to the bucket of zero (+ answer) or taking sets of -4 from the bucket of zero (- answer)?

Solution process:

To make the value of the bucket -8 I would need to take away two sets of -4, so my solution must be -2.



$$(-8) \div (+2) = ? -4$$

Think:

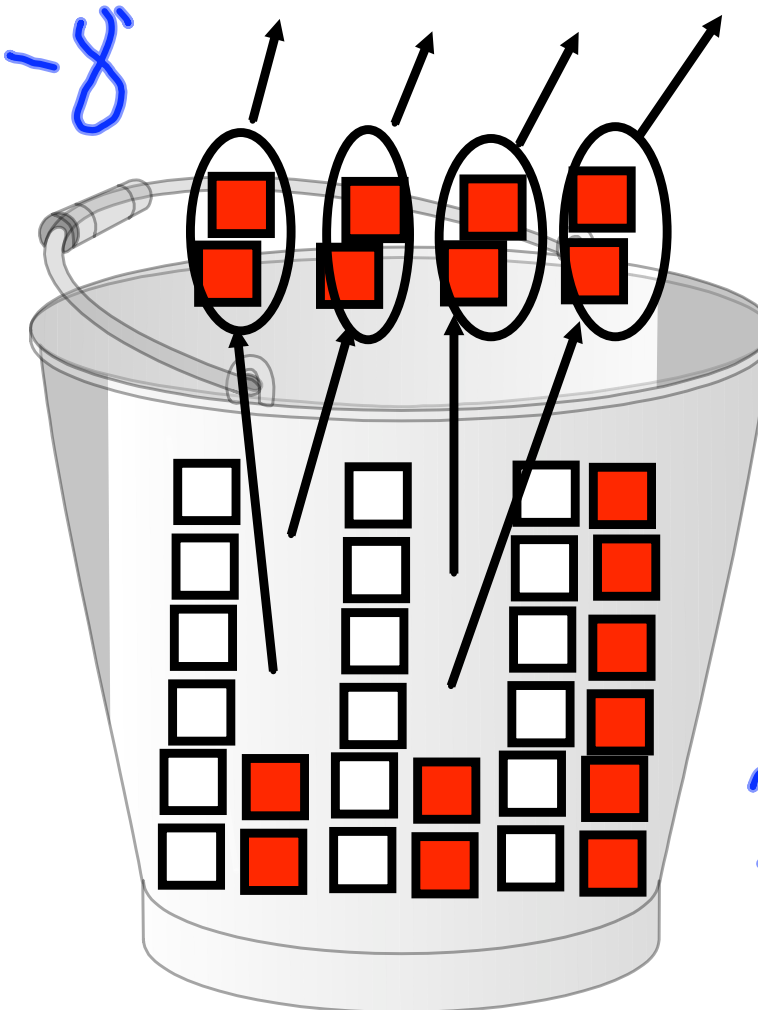
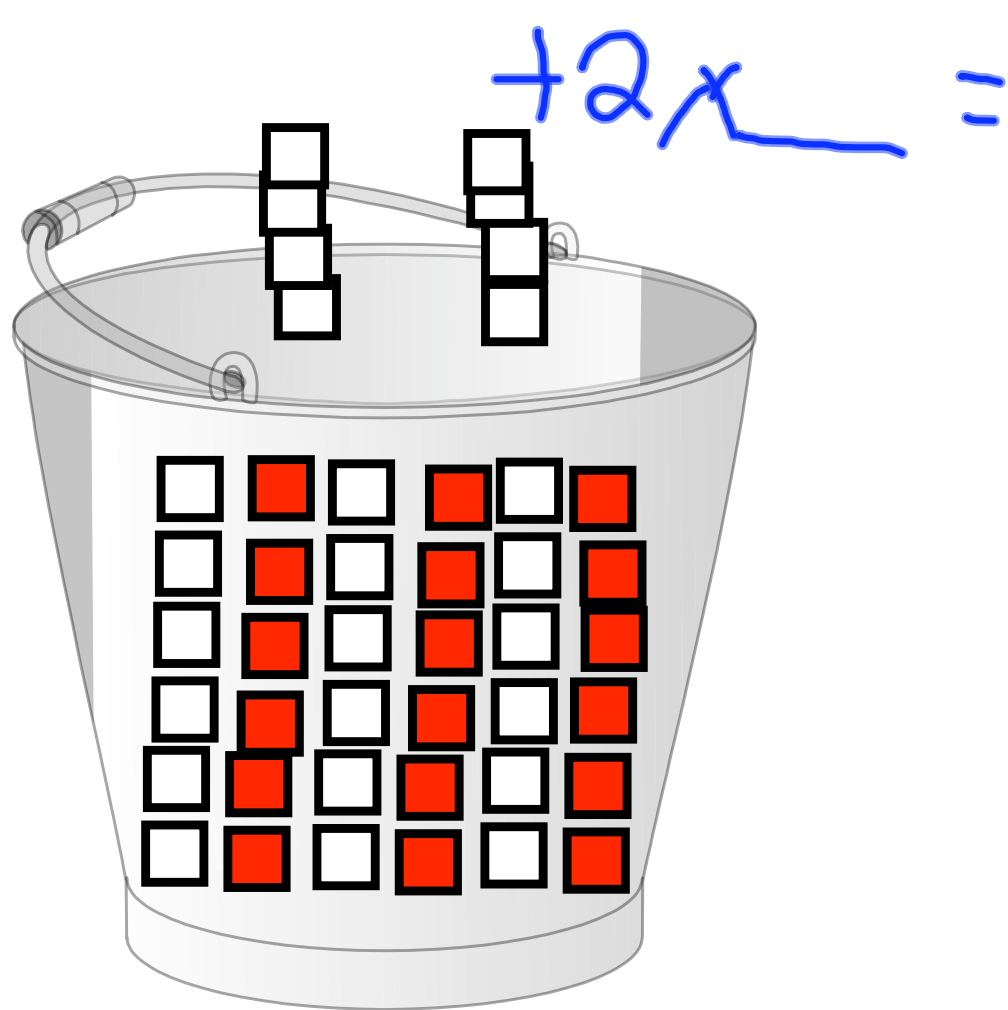
How many sets of +2 do I need to make -8?
 Will I be adding sets of +2 to the bucket of zero (+ answer) or taking sets of +2 from the bucket of zero (- answer)?

$$(-8) \div (+2) = (-4)$$



Solution process:

To make the value of the bucket -8 I would need to take away four sets of +2 so my solution must be -4.

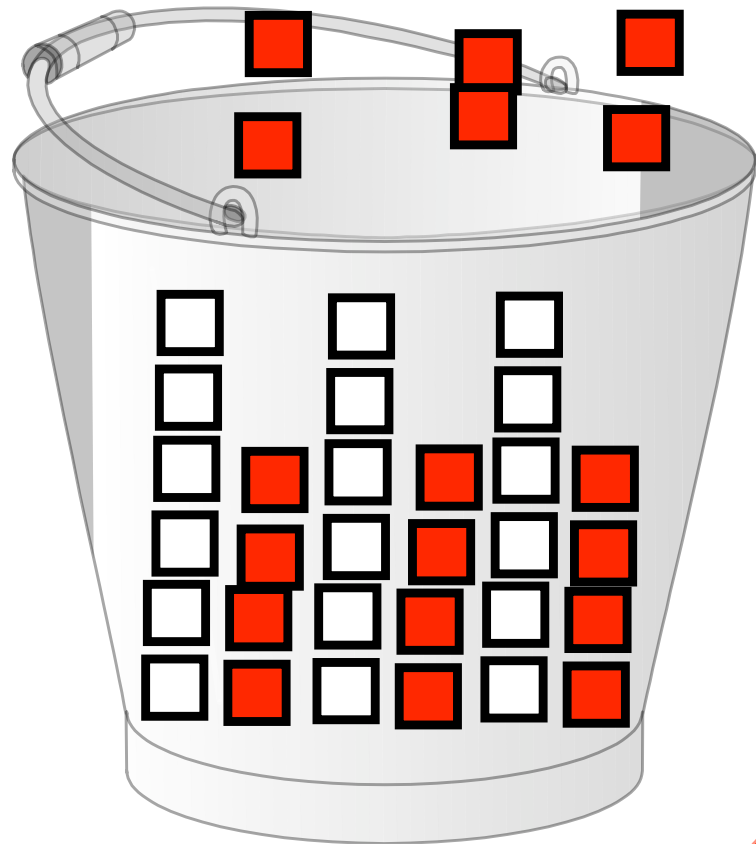


$$-4 \times +2 = -8$$

$$(-6) \div (-3) = ?$$

Think:

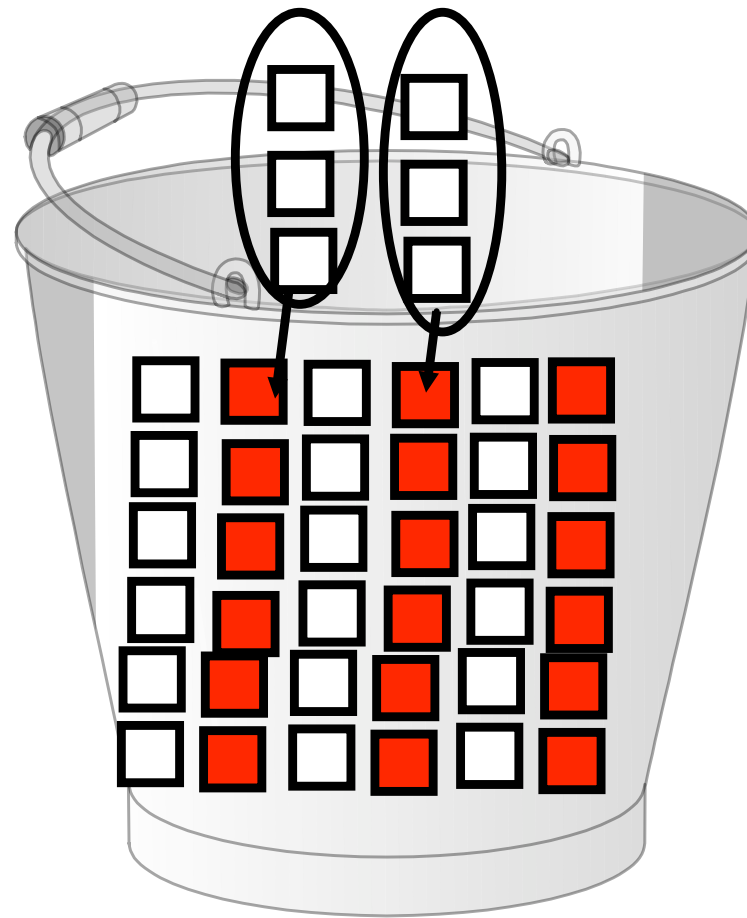
How many sets of -3 do I need to make -6?
Will I be adding sets of -3 to the bucket of zero (+ answer) or taking sets of -3 from the bucket of zero (- answer)?



$$(-6) \div (-3) = (+2)$$

Solution process:

To make the value of the bucket -6 I would need to add two sets of -3, so my solution must be +2.

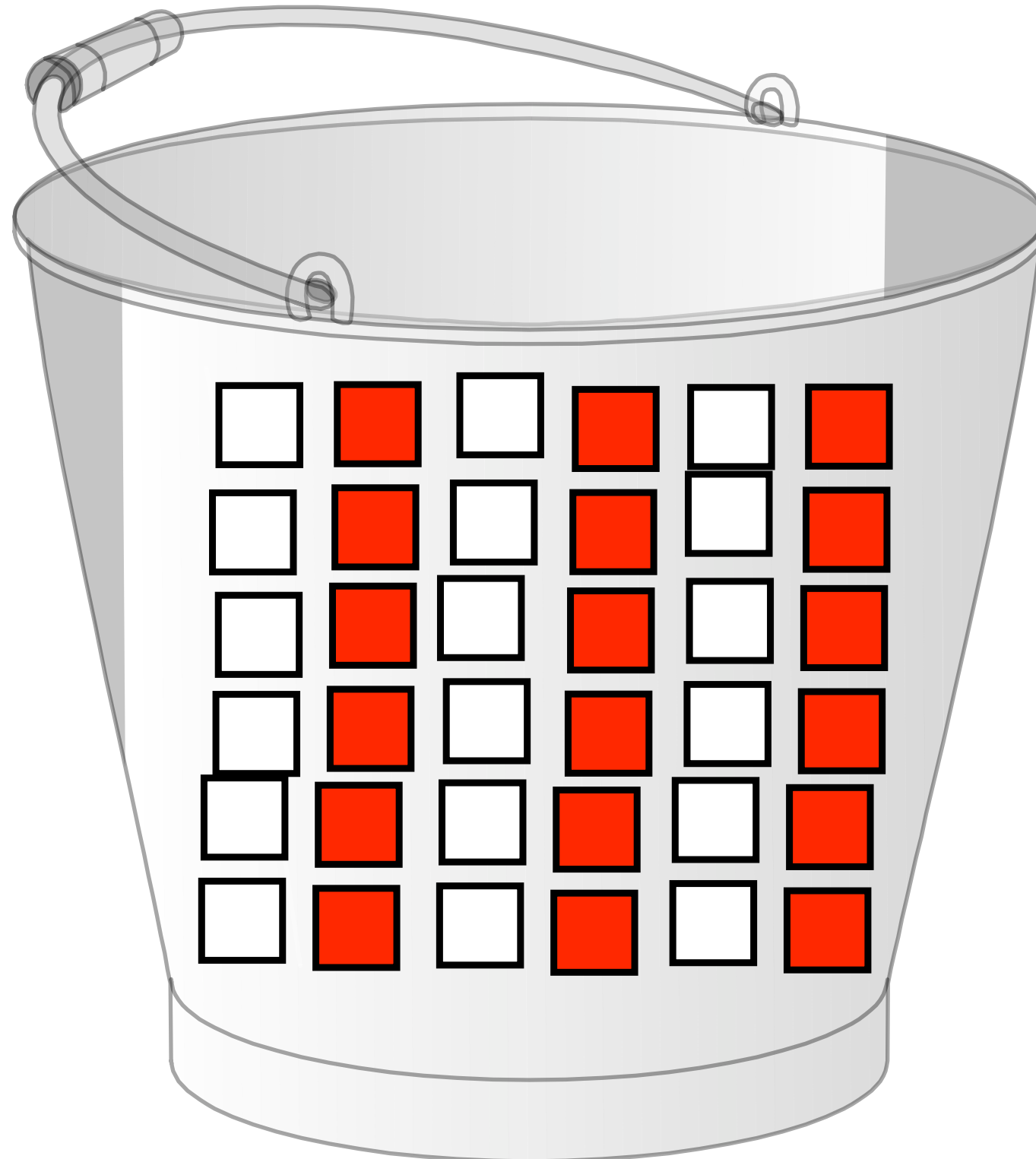


$$+2 \times -3 = -6 \quad -3 \times +2 = -6$$

Say it in words:

How do I make the bucket have a value of -15 if I can only use groups of -3?

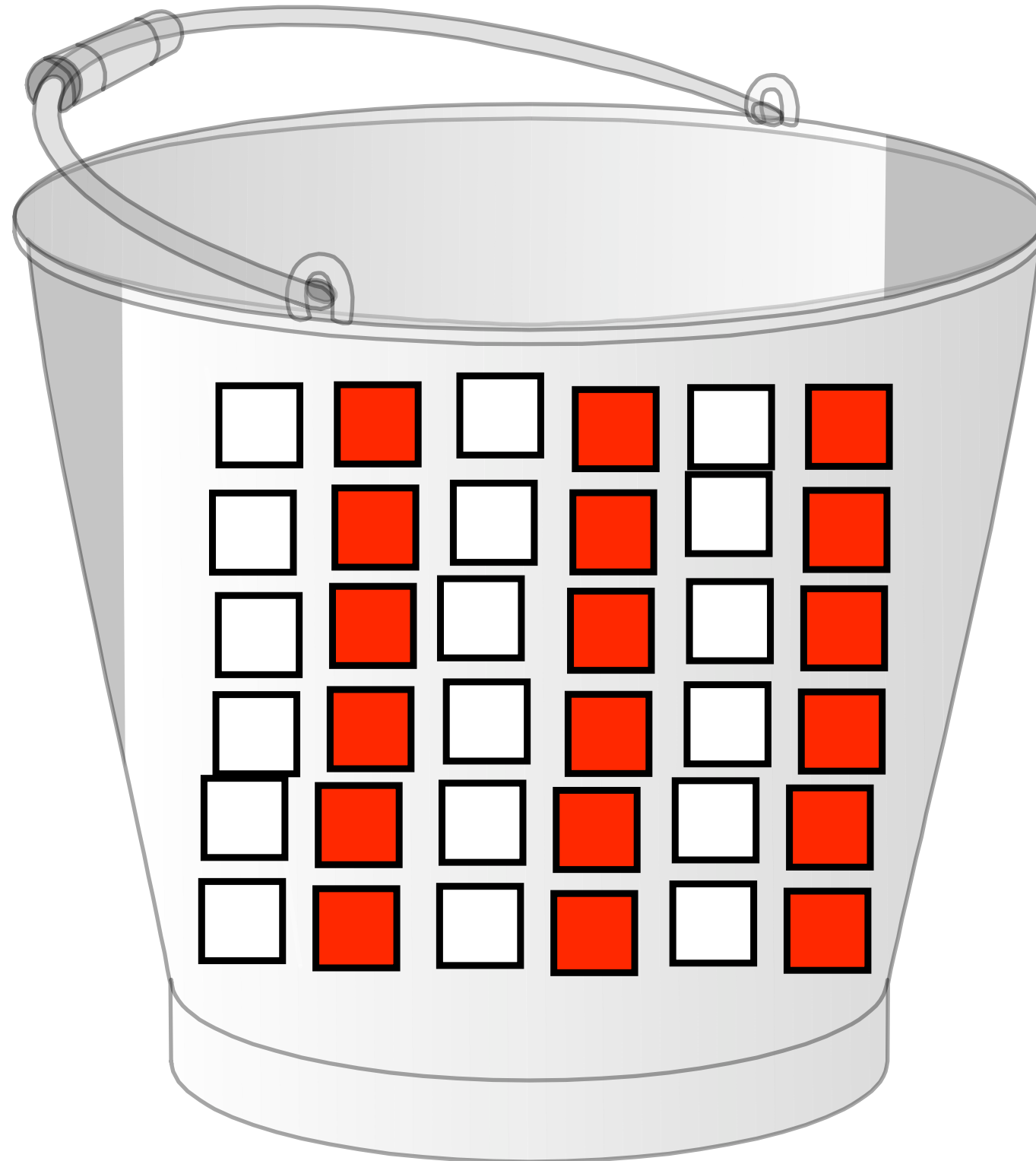
$$(-15) \div (-3)$$



Say it in words:

How do I make the bucket have a value of -12 if I can only use groups of +3?

$$(-12) \div (+3)$$

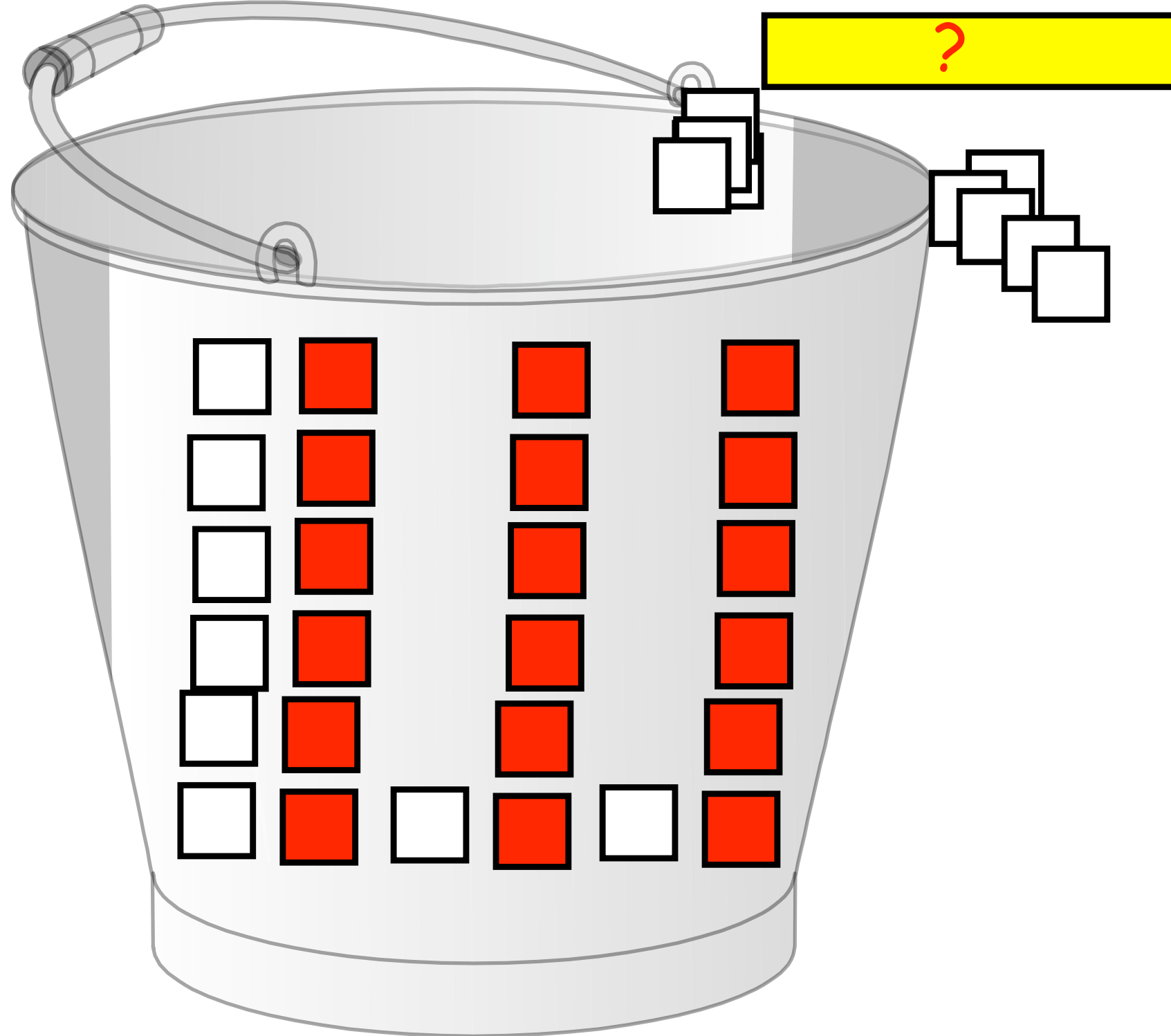
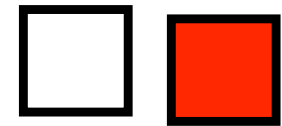


Say it in words:

How do I make the bucket have a value of +10 if I can only use groups of -2?

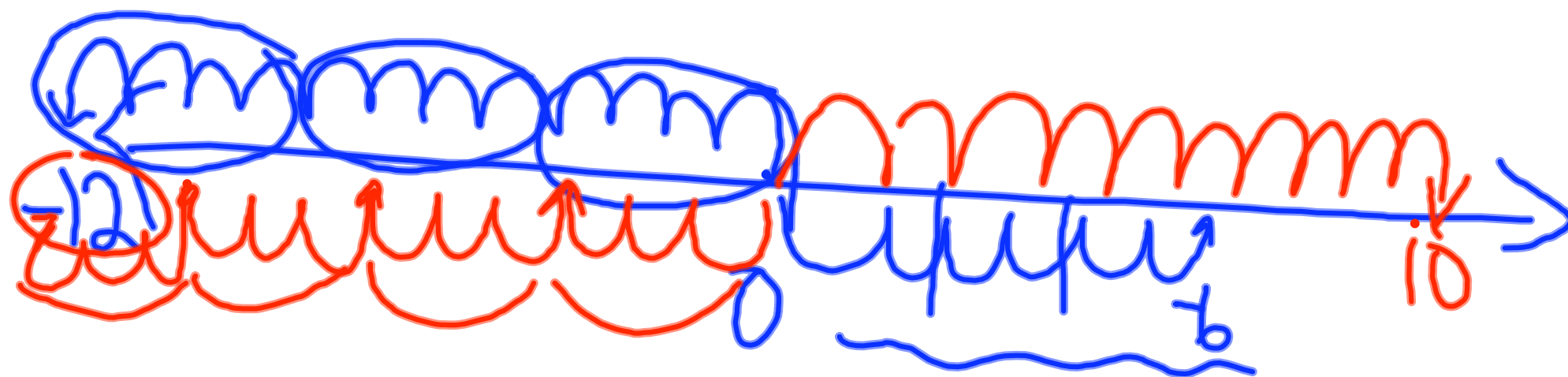
$$(+10) \div (-2)$$

$$(+10) \div (-2) = -5$$



$$\underline{+3x - 4}$$

$$\begin{aligned} & -4x + 3 \\ & +2x + 5 \end{aligned}$$



$$-3x - 2$$